ERRATUM

Z. P. Huang

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The corresponding author of the above paper regrets that in the expressions of the first kind Piola–Kirchhoff surface stress $\mathbf{S}_s$ and the Lagrangian description of the Young–Laplace equations some terms associated with the “out-plane term” of surface gradient $\mathbf{F}_s$ and “out-plane term” of $\mathbf{S}_s$ were missing. In fact, $\mathbf{F}_s$ and $\mathbf{S}_s$ are “two-point” tensors, with base vectors both on the tangent planes $\mathcal{J}_Y^0$ and $\mathcal{J}_y$, and are not tensors in the two-dimensional tangent plane $\mathcal{J}_Y^0$ only. The correct forms of $\mathbf{F}_s$, $\mathbf{S}_s$ and the Lagrangian description of the Young–Laplace equations are given as follows.

(i) If the displacement on the surface $\mathbf{u}$ is decomposed into $\mathbf{u}_{0s} = u_0^\alpha \mathbf{A}_\alpha$ in the tangent plane $\mathcal{J}_Y^0$ and $\mathbf{u}_{0n} = u_0^n \mathbf{A}_3$ along the normal direction of $\mathcal{J}_Y^0$, then the surface gradient can be written as

$$\mathbf{F}_s = \mathbf{a}_\alpha \otimes \mathbf{A}^\alpha = \mathbf{i}_0 + \mathbf{u} \nabla_{0s} + d_{0\alpha} \mathbf{A}_3 \otimes \mathbf{A}^\alpha \quad (\text{c1})$$

where $\mathbf{i}_0$ is the second-order identity tensor on $\mathcal{J}_Y^0$, $\mathbf{u} \nabla_{0s}$ is defined as

$$\mathbf{u} \nabla_{0s} = \mathbf{u}_{0s} \nabla_{0s} - u_0^n \mathbf{b}_0 = u_0^\lambda |_\alpha \mathbf{A}_\lambda \otimes \mathbf{A}^\alpha - u_0^n \mathbf{b}_0 \quad (\text{c2})$$

and $d_{0\alpha} = u_0^\lambda b_{0\lambda\alpha} + u_0^n |_\alpha$, $\mathbf{b}_0 = b_{0\lambda\alpha} \mathbf{A}^\lambda \otimes \mathbf{A}^\alpha$ is the curvature tensor of the surface in the reference configuration.

Equation (c1) can be expressed as the sum of an “in-plane term” $\mathbf{F}_s^{(in)} = \mathbf{i}_0 + \mathbf{u} \nabla_{0s}$ and an “out-plane term” $\mathbf{F}_s^{(ou)} = d_{0\alpha} \mathbf{A}_3 \otimes \mathbf{A}^\alpha$. Hence the first kind Piola–Kirchhoff surface stress can also be decomposed into an “in-plane term” $\mathbf{S}_s^{(in)} = \mathbf{F}_s^{(in)} \cdot \mathbf{T}_s^{(1)}$, and an “out-plane term” $\mathbf{S}_s^{(ou)} = \mathbf{F}_s^{(ou)} \cdot \mathbf{T}_s^{(1)}$.

(ii) The “out-plane term” $\mathbf{S}_s^{(ou)}$ in the above paper is missing. In Eq. (4), this missing term is $\mathbf{S}_s^{(ou)} = J_2 (\partial \gamma / \partial J_1 + J_2 \partial \gamma / \partial J_2 + \gamma) \mathbf{F}_s^{(ou)}$, and in Eqs. (12) and (15), this missing term is $\gamma_0^* \mathbf{F}_s^{(ou)}$, and the discussion on Eq. (16) is inappropriate due to this missing term.

In Eq. (6), the missing term is $(\partial \gamma / \partial J_1 + J_2 \partial \gamma / \partial J_2 + \gamma) \left(\mathbf{F}_s^{(ou)} + \mathbf{F}_s^{(ou)T} \right)$, and therefore, the missing term in Eq. (13) is $\gamma_0^* \left(\mathbf{F}_s^{(ou)} + \mathbf{F}_s^{(ou)T} \right)$. It should be mentioned that, for small deformation, $\mathbf{i}_0 + \left(\mathbf{F}_s^{(ou)} + \mathbf{F}_s^{(ou)T} \right)$ is

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Z. P. Huang (✉)
LTCS and Department of Mechanics and Aerospace Engineering,
College of Engineering, Peking University, Beijing 100871, China
E-mail: huangzp@pku.edu.cn

equal to the second-order identity tensor $\mathbf{i}$ in the tangent plane $\mathcal{J}_y$, and the Cauchy surface stress is a tensor in tangent plane $\mathcal{J}_y$ of the deformed surface.

Equation (7) should be written as

$$\begin{aligned}\mathbf{N} \cdot [\mathbf{S}^0] \cdot \mathbf{N} &= -(\mathbf{S}_s^{(in)}) : \mathbf{b}_0 - [\mathbf{N} \cdot (\mathbf{S}_s^{(ou)})] \cdot \nabla_{0s}, \\ \mathbf{P}_0 \cdot [\mathbf{S}^0] \cdot \mathbf{N} &= -(\mathbf{S}_s^{(in)}) \cdot \nabla_{0s} + [\mathbf{N} \cdot (\mathbf{S}_s^{(ou)}) \cdot \mathbf{b}_0]\end{aligned}\quad (\text{c3})$$

and accordingly Eq. (8) should be

$$\begin{aligned}\mathbf{N} \cdot [\Delta \mathbf{S}^0] \cdot \mathbf{N} &= -(\Delta \mathbf{S}_s^{(in)}) : \mathbf{b}_0 - [\mathbf{N} \cdot (\Delta \mathbf{S}_s^{(ou)})] \cdot \nabla_{0s}, \\ \mathbf{P}_0 \cdot [\Delta \mathbf{S}^0] \cdot \mathbf{N} &= -(\Delta \mathbf{S}_s^{(in)}) \cdot \nabla_{0s} + [\mathbf{N} \cdot (\Delta \mathbf{S}_s^{(ou)}) \cdot \mathbf{b}_0]\end{aligned}\quad (\text{c4})$$

Equation (27) should be

$$\begin{aligned}\llbracket \sigma_{rr} \rrbracket|_{r=a} &= \frac{1}{a^2} (\gamma_0^* + 2\gamma_1^* + \gamma_1) \left(2u_r + u_\theta \cot \theta + \frac{\partial u_\theta}{\partial \theta} \right) \Big|_{r=a} \\ &\quad - \frac{1}{a^2} \gamma_0^* \left(\frac{\partial^2 u_r}{\partial \theta^2} - \frac{\partial u_\theta}{\partial \theta} + \frac{\partial u_r}{\partial \theta} \cot \theta - u_\theta \cot \theta \right) \Big|_{r=a}, \\ \llbracket \sigma_{r\theta} \rrbracket|_{r=a} &= \frac{1}{a^2} \left[(2\gamma_0^* + \gamma_1^*) u_\theta + (\gamma_1^* + \gamma_1) \left(u_\theta \cot^2 \theta - \frac{\partial u_\theta}{\partial \theta} \cot \theta - \frac{\partial^2 u_\theta}{\partial \theta^2} \right) \right] \Big|_{r=a} \\ &\quad - \frac{1}{a^2} \left[(2\gamma_0^* + 2\gamma_1^* + \gamma_1) \frac{\partial u_r}{\partial \theta} \right] \Big|_{r=a}.\end{aligned}\quad (\text{c5})$$

Equation (34) should be

$$\mu_* = \mu_1 + \frac{L_1 + L_2}{L_3 + L_4} \quad (\text{c6})$$

where

$$\begin{aligned}L_0 &= 2\gamma_0^{*2} - \gamma_1 (\gamma_1 + 2\gamma_1^*) - \gamma_0^* (5\gamma_1 + 2\gamma_1^*), \quad L_1 = 10(7 - 10\nu_1) L_0, \\ L_2 &= -\frac{1}{10a} (2\gamma_0^* - 7\gamma_1 - 2\gamma_1^*) L_4, \quad L_3 = 4(-7 + 10\nu_1) a (13\gamma_0^* + 7\gamma_1 + 12\gamma_1^*), \\ L_4 &= 10a^2 [4(-7 + 10\nu_1) \mu_0 - (7 + 5\nu_1) \mu_1].\end{aligned}$$

Based on (c6), Eq. (36) should be written as

$$\bar{\mu} = \mu_0 + \frac{15\mu_0 f (1 - \nu_0) [-L_1 - L_2 + (L_3 + L_4) (\mu_0 - \mu_1)]}{L_5 + L_6} \quad (\text{c7})$$

where

$$\begin{aligned}L_5 &= 2(1 - f)(-4 + 5\nu_0)(L_1 + L_2), \\ L_6 &= (L_3 + L_4)[(-7 + 5\nu_0)\mu_0 + 2(-4 + 5\nu_0)(f\mu_0 + \mu_1 - f\mu_1)].\end{aligned}$$

Therefore, Eq. (38) should be

$$\bar{\mu}_{\text{void}} = \mu_0 + 15\mu_0 f (1 - \nu_0) \frac{L_0 + M_1 + M_2}{N_0 + N_1 + N_2} \quad (\text{c8})$$

where

$$\begin{aligned}M_1 &= 2(3\gamma_0^* + 2\gamma_1^*) \mu_0 a, \quad M_2 = 4\mu_0^2 a^2, \quad N_0 = 2(1 - f)(4 - 5\nu_0) L_0, \\ N_1 &= 2[3N\gamma_0^* + 21(-1 + \nu_0)\gamma_1 + 2(N - 5 + 5\nu_0)\gamma_1^*] \mu_0 a, \\ N_2 &= 4a^2 (N - 2 + 2\nu_0) \mu_0^2, \quad N = -5 + 3\nu_0 + 2f(-4 + 5\nu_0).\end{aligned}$$

From (c7) with $\gamma_1 = \gamma_1^* = 0$, Eq. (40) can be corrected easily.

It is noted that the basic idea, theoretical framework and main results presented in the paper are not affected by these missing terms.

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