Z. P. Huang · J. Wang

Erratum to: A theory of hyperelasticity of multi-phase media with surface/interface energy effect

Published online: 15 October 2010
© Springer-Verlag 2010

Erratum to: Acta Mechanica 182, 195–210 (2006)
DOI 10.1007/s00707-005-0286-3

We regret that in the expressions of the first kind Piola-Kirchhoff surface stress $\mathbf{S}_s$ and the Lagrangian description of the Young-Laplace equations (Eqs. 35–36), some terms associated with the “out-plane term” of surface gradient $\mathbf{F}_s$ and “out-plane term” of $\mathbf{S}_s$ were missing. In fact, $\mathbf{F}_s$ and $\mathbf{S}_s$ are “two-point” tensors, with base vectors both on the tangent planes $\mathcal{J}_Y^0$ and $\mathcal{J}_y$, and are not tensors in the 2-dimensional tangent plane $\mathcal{J}_Y^0$ only. The correct forms of $\mathbf{F}_s$, $\mathbf{S}_s$ and the Lagrangian description of the Young-Laplace equations are given as follows.

(1) If the displacement on the surface $\mathbf{u}$ is decomposed into $\mathbf{u}_{0s} = u_0^\alpha \mathbf{A}_\alpha$ in the tangent plane $\mathcal{J}_Y^0$ and $\mathbf{u}_{0n} = u_0^n \mathbf{A}_3$ along the normal direction of $\mathcal{J}_Y^0$, then the surface gradient can be written as

$$\mathbf{F}_s = \mathbf{a}_\alpha \otimes \mathbf{A}^\alpha = \mathbf{i}_0 + \mathbf{u} \nabla_{0s} + d_{0\alpha} \mathbf{A}_3 \otimes \mathbf{A}^\alpha, \quad (\text{c1})$$

where $\mathbf{i}_0$ is the second-order identity tensor on $\mathcal{J}_Y^0$, $\mathbf{u} \nabla_{0s}$ is defined as

$$\mathbf{u} \nabla_{0s} = \mathbf{u}_{0s} \nabla_{0s} - u_0^n \mathbf{b}_0 = u_0^\lambda |_\alpha \mathbf{A}_\lambda \otimes \mathbf{A}^\alpha - u_0^n \mathbf{b}_0 \quad (\text{c2})$$

and $d_{0\alpha} = u_0^\lambda b_{0\lambda\alpha} + u_0^n |_\alpha$, $\mathbf{b}_0 = b_{0\lambda\alpha} \mathbf{A}^\lambda \otimes \mathbf{A}^\alpha$ is the curvature tensor of the surface in the reference configuration.

Equation (c1) can be expressed as the sum of an “in-plane term” $\mathbf{F}_s^{(in)} = \mathbf{i}_0 + \mathbf{u} \nabla_{0s}$, and an “out-plane term” $\mathbf{F}_s^{(ou)} = d_{0\alpha} \mathbf{A}_3 \otimes \mathbf{A}^\alpha$. Hence, the first kind Piola-Kirchhoff surface stress can also be decomposed into an “in-plane term” $\mathbf{S}_s^{(in)} = \mathbf{F}_s^{(in)} \cdot \mathbf{T}_s^{(1)}$, and an “out-plane term” $\mathbf{S}_s^{(ou)} = \mathbf{F}_s^{(ou)} \cdot \mathbf{T}_s^{(1)}$.

(2) $\delta \mathbf{u} \nabla_0$ and $\delta \mathbf{u} \nabla$ on the right-hand side of Eq. (21) (and hence on the left-hand side of Eqs. 22 and 23) should be replaced by $\delta \mathbf{a}_\alpha \otimes \mathbf{A}^\alpha = \delta \mathbf{F}_s$ and $\delta \mathbf{a}_\alpha \otimes \mathbf{a}^\alpha$, respectively.

Equation (22) should be

$$\begin{aligned} \mathbf{S}_s : \delta \mathbf{F}_s = & \mathbf{S}_s^{(in)} : \delta \mathbf{F}_s^{(in)} + \mathbf{S}_s^{(ou)} : \delta \mathbf{F}_s^{(ou)} = \left(\delta \mathbf{u}_{0s} \cdot \mathbf{S}_s^{(in)} \right) \cdot \nabla_{0s} + \left(\delta u_0^n \mathbf{A}_3 \cdot \mathbf{S}_s^{(ou)} \right) \cdot \nabla_{0s} \\ & - \delta \mathbf{u}_{0s} \cdot \left(\mathbf{S}_s^{(in)} \cdot \nabla_{0s} - \mathbf{b}_0 \cdot \mathbf{S}_s^{(ou)T} \cdot \mathbf{A}_3 \right) - \delta u_0^n \left(\mathbf{S}_s^{(in)} : \mathbf{b}_0 + \left(\mathbf{A}_3 \cdot \mathbf{S}_s^{(ou)} \right) \cdot \nabla_{0s} \right). \end{aligned} \quad (\text{c3})$$

The online version of the original article can be found under doi:10.1007/s00707-005-0286-3.

Z. P. Huang (✉) · J. Wang
LTCS and Department of Mechanics and Aerospace Engineering, College of Engineering,
Peking University, 100871 Beijing, China
E-mail: huangzp@pku.edu.cn

$\sigma_s : (\delta \mathbf{u} \nabla)$ on the left hand side of Eqs. (23) and (27) should be $\sigma_s : (\delta \mathbf{a}_\alpha \otimes \mathbf{a}^\alpha) = \sigma_s : (\delta \mathbf{u} \nabla_s)$, where $\delta \mathbf{u} \nabla_s = \delta u^\alpha |_\beta \mathbf{a}_\alpha \otimes \mathbf{a}^\beta - \delta u^n \mathbf{b}$.

Equation (26) should be

$$\begin{aligned} \int_{A_0} \mathbf{S}_s : \delta \mathbf{F}_s dA_0 = & - \int_{\partial \Omega_0} \delta \mathbf{u}_{0s} \cdot \left[\left[\mathbf{S}_s^{(in)} \right] \right] \cdot \mathbf{n}_0 dl_0 - \int_{\partial \Omega_0} \delta u_0^n \mathbf{A}_3 \cdot \left[\left[\mathbf{S}_s^{(ou)} \right] \right] \cdot \mathbf{n}_0 dl_0 \\ & + \int_{A_0} \delta \mathbf{u}_{0s} \cdot \left(- \mathbf{S}_s^{(in)} \cdot \nabla_{0s} + \mathbf{b}_0 \cdot \mathbf{S}_s^{(ou)T} \cdot \mathbf{A}_3 \right) dA_0 \\ & - \int_{A_0} \delta u_0^n \left(\mathbf{S}_s^{(in)} : \mathbf{b}_0 + \left(\mathbf{A}_3 \cdot \mathbf{S}_s^{(ou)} \right) \cdot \nabla_{0s} \right) dA_0. \end{aligned} \quad (c4)$$

Equations (35) and (36) should be written as

$$\begin{aligned} \mathbf{A}_3 \cdot \left[\left[\mathbf{S}^0 \right] \right] \cdot \mathbf{A}_3 = & - \left(\mathbf{S}_s^{(in)} \right) : \mathbf{b}_0 - \left[\mathbf{A}_3 \cdot \left(\mathbf{S}_s^{(ou)} \right) \right] \cdot \nabla_{0s}, \\ \mathbf{P}_0 \cdot \left[\left[\mathbf{S}^0 \right] \right] \cdot \mathbf{A}_3 = & - \left(\mathbf{S}_s^{(in)} \right) \cdot \nabla_{0s} + \left[\mathbf{A}_3 \cdot \left(\mathbf{S}_s^{(ou)} \right) \cdot \mathbf{b}_0 \right]. \end{aligned} \quad (c5)$$

It is noted that the basic idea, theoretical framework, and main results presented in the paper are not affected by these missing terms.