Erratum to: The G_2 sphere of a 4-manifold

R. Albuquerque · I. Salavessa

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The article in question presents a universal construction of a G_2 structure on the unit sphere tangent bundle S_M of any given orientable Riemannian 4-manifold. The G_2 structure 3-form $\phi = \alpha + \mu\beta - \alpha_2$ and 4-form $*\phi = f^*\text{vol}_M - \frac{1}{2}\beta^2 - \mu\alpha_1$ were deduced and their exterior derivatives were computed. However, relevant components of the involved $d\alpha_1$ and $d\alpha_2$ were missed by the authors for quite long, most fortunately not affecting the main results. The hidden tensors will become obvious to the reader by the end of this note. We start by looking at $D_X\alpha(Y_1, Y_2, Y_3)$ in the proof of Proposition 3.2. It has a component

$$f^{-1}\text{vol}_M(\nabla_X^*U, Y_1, Y_2, Y_3) = f^{-1}\text{vol}_M(X^v, Y_1, Y_2, Y_3)$$

which surely vanishes because S_M does not admit four linearly independent vertical vector fields, ie. tangent to the S^3 -fibres. However, in computing $D\alpha_i = \frac{1}{2}D(\alpha \circ \theta^i \wedge 1)$, $i = 1, 2$, then θ may pick the horizontal vector field $\theta^i U$, send it to U and thence

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R. Albuquerque (✉)
Departamento de Matemática da Universidade de Évora and Centro de Investigação
em Matemática e Aplicações (CIMA),
Rua Romão Ramalho, 59, 671-7000 Évora, Portugal
e-mail: rpa@dmate.uevora.pt

I. Salavessa
Centro de Física das Interações Fundamentais (CFIF), Instituto Superior Técnico,
Av. Rovisco Pais, Edifício Ciência, Piso 3, 1049-001 Lisbon, Portugal
e-mail: isabel.salavessa@ist.utl.pt

add a non despising value through the referred component. This changes the result of Proposition 3.2, item 2, and implies, as it is easy to deduce, that one must add

$$3\mu \wedge \alpha \quad \text{and} \quad 2\mu \wedge \alpha_1$$

to, respectively, $d\alpha_1$ and $d\alpha_2$. Therefore we must subtract $2\mu \wedge \alpha_1$ in formula (20) and its proof, in Theorem 3.1, and subtract $3\mu \wedge \alpha$ in that of Proposition 3.3. Yet Corollary 3.1 remains true. Just as well as Proposition 3.3 is correct. We may reassure the reader that S_M is co-calibrated if, and only if, M is an Einstein manifold. Indeed, notice $d * \phi = \mu \wedge d\alpha_1$ and $\mu \wedge (3\mu \wedge \alpha) = 0$, hence the same exact expression for $\delta\phi$ is achieved. Moreover, all qualitative results and observations throughout remain unchanged.

Now, in all the results for the values of $d\phi$ in formulae (23), (24) and (25), as explained above, we must subtract

$$2\mu \wedge \alpha_1 = -\frac{6}{7} * \phi + \frac{1}{7} (6\alpha + 6\mu\beta + 8\alpha_2).$$

On the right hand side we have the decomposition into G_2 -irreducible components, which influence τ_0 and τ_3 accordingly with (22).