Arkadiusz P. Trawiński · Stanisław D. Głazek

Example of a Model for AdS/QFT Duality

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Abstract Models of hadrons that are rooted in light-front (LF) formulation of QCD have been linked to the classical field equations in a 5-dimensional anti-de Sitter (AdS) gravitational background in terms of the Brodsky-de Téramond LF holography. We discuss the classical equations of motion for the expectation values of operators in quantum field theory whose nature resembles the Ehrenfest equations of quantum mechanics and which thus appear to provide a general justification for the holographic picture. The required expectation values are obtained by distinguishing one effective constituent of a hadron, the one that is struck by an external electro-weak or gravitational probe, and integrating over relative motion variables of all other constituents in all Fock components. The scale-dependent Fock decomposition of hadronic states is defined using the renormalization group procedure for effective particles. The AdS modes dual to the incoming and outgoing hadrons in the corresponding transition matrix elements are thus found equivalent to the Gaussian form distribution functions for the effective partons struck by external probes.

1 Introduction

Perturbative QCD successfully describes high-energy collisions of hadrons using parton models [1], but encounters difficulties with explaining physics of hadrons understood as bound states of quarks [2,3] as far as observables that are not accessible using Euclidean lattice formulation of QCD [4] are concerned. The Maldacena conjecture of AdS/CFT duality [5] has been used by Polchinsky and Strassler [6,7] to argue that hadrons may be described in terms of classical fields in a five-dimensional space-time, hopefully providing a initial approximation to the physics of hadrons. de Téramond and Brodsky [8,9] discovered that the Polchinski and Strassler formulae for form factors of mesons are precisely matched by the corresponding formulae in the light-front (LF) formulation of quantum field theory (QFT) when one identifies the differential equation in 5th dimension for a meson field with the transverse radial eigenvalue equation for a meson valence wave function in the LF Fock space. In addition, the equation of motion in AdS is identical to the LF Hamiltonian eigenvalue equation for a bound state of two constituents. This matching is called LF holography. The LF holography has been extended to baryons in terms of the transverse radial quark–diquark wave functions.

In this paper we discuss an example which shows that the Ehrenfest correspondence principle between quantum and classical mechanics [10] can be adapted to provide, in combination with the LF holography, a natural explanation of how a complex QFT could reduce to the simple dual picture [11].

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A. P. Trawiński (✉) · S. D. Głazek
Faculty of Physics, Institute of Theoretical Physics, University of Warsaw, Warsaw, Poland
E-mail: Arkadiusz.Trawinski@fuw.edu.pl

S. D. Głazek
E-mail: Stanislaw.Glazek@fuw.edu.pl

We assume that the front form (FF) of Hamiltonian dynamics [12] yields a representation of eigenstates that correspond to hadrons in terms of a suitably defined effective Fock-space expansion once the theory is renormalized using the renormalization group procedure for effective particles (RGPEP) [13]. The RGPEP framework is close in its principles to the similarity renormalization group procedure originally introduced in Ref. [14] and used in Ref. [15]. It has the advantage of the non-perturbative operator calculus recently described in Refs. [16, 17] in terms of elementary examples.

For all tensors, we use the same convention as for the position co-ordinates $x^\pm = x^0 \pm x^3$ and $x^\perp = (x^1, x^2)$, which is customary for the front defined by $x^+ = 0$. The eigenstate $|\text{Hadron} : P^+, P^\perp\rangle$ of a FF QFT Hamiltonian $\hat{P}^-$, being represented at the RGPEP scale λ as a collection of Fock components labeled by a number n of effective constituents corresponding to λ with λ -dependent wave functions $\psi_P^{(n)}$, reads

$$|\text{Hadron} : P^+, P^\perp\rangle = \sum_{n=1}^{\infty} \int [p^\perp, \mathbf{x}] \psi_P^{(n)}(p^\perp, \mathbf{x}; \lambda) |n : p^\perp, \mathbf{x} P^+; \lambda\rangle, \quad (1)$$

where the transverse momenta of the constituents are denoted by $p^\perp = (p_i^\perp)_{i=1, \dots, n}$ and ratios of longitudinal momenta of constituents to the hadron longitudinal momentum are denoted by $\mathbf{x} = (x_i)_{i=1, \dots, n}$, $x_i = p_i^+ / P^+$. The symbol $\int [p^\perp, \mathbf{x}]$ denotes integration over $p^\perp$ and $\mathbf{x}$ with the relativistic weight $1/[2x(2\pi)^3]$ for each and every constituent. The eigenvalue equation is

$$\hat{P}^- |\text{Hadron} : P^+, P^\perp\rangle = \frac{M^2 + P^{\perp 2}}{P^+} |\text{Hadron} : P^+, P^\perp\rangle, \quad (2)$$

where the eigenvalue is expressed using the kinematical components P^+ and $P^\perp$ of the total momentum of the system and its mass squared, M^2 .

2 Example of a QFT-Model for a Hadron State

The example described below requires a specification of a basis in the Fock space and a definition of the corresponding wave functions, both defined at some scale λ . The simplest model we can imagine can be built assuming that all constituents are all scalar boson quanta of one kind. Thus, we assume that the basis states $|n : p^\perp, \mathbf{x} P^+; \lambda\rangle$ in Eq. (1) are of the form

$$|n : p^\perp, \mathbf{x} P^+; \lambda\rangle = \frac{1}{\sqrt{n!}} \prod_{i=1}^n a^\dagger(p_i^\perp, x_i P^+; \lambda) |0\rangle. \quad (3)$$

The corresponding wave functions $\psi_P^{(n)}(p^\perp, \mathbf{x}; \lambda)$ in Eq. (1) are fully symmetric functions of their arguments under permutations of numbers $i = 1, \dots, n$.

We assume that each constituent in every Fock component is bound by some effective interaction at scale λ to other constituents in the same Fock sector. In every sector, we distinguish one constituent that is named active, and we focus on description of its motion with respect to other constituents. The other constituents are named spectators or a core, depending on the context. Thus, we expect that a reasonable model is obtained by assuming that every active constituent is attracted to a minimum of the effective potential energy in its Fock sector. Around this minimum, the effective potential in every Fock component is assumed to be a quadratic function of the distance between the active constituent and the center of mass of spectators. The imagined situation in a Fock sector resembles the Thomson model of an atom [18], in which an electron is attracted to the center of a distribution of a positive charge, or the model of a nucleus where a single nucleon moves in a potential created by other nucleons. Our model differs from these examples by the new element which is the averaging over the presumably infinitely large collection of effective Fock components, instead of just one state with a fixed number of constituents. Our constituent quanta are defined using the RGPEP. Only the coherent collection of all the Fock sectors satisfies the FF Hamiltonian eigenvalue equation for the whole hadron.

Fig. 1 Transverse position-coordinates of constituents in one Fock sector. The absolute transverse position of i th constituent is denoted by $r_i^\perp$. Transverse position of the whole hadron is denoted by $R^\perp = \sum_{i=1}^n x_i r_i^\perp$ (cf. Ref. [19]). Position of the center of mass of the spectators accompanying the constituent i is defined by $R_i^\perp = \sum_{j \neq i} x_j r_j^\perp / \sum_{l \neq i} x_l$. We denote by $\eta_i^\perp$ the relative distance between the i th constituent and the hadron position, $\eta_i^\perp = r_i^\perp - R^\perp = (1 - x_i)(r_i^\perp - R_i^\perp)$. The latter equality says that $\eta_i^\perp$ is proportional to the transverse distance between the i th constituent and the center of mass of the spectators

The motion of the active constituent can be described using its transverse position with respect to the center of mass of the hadron, illustrated by $\eta_i^\perp$ in Fig. 1. If the effective one-particle binding potential is harmonic, the wave function in the Fock component of n effective constituents is then precisely of the form

$$\tilde{\psi}^{(n)}(\eta^\perp, \mathbf{x}; \lambda) = \kappa_n^n \tilde{A}_n(\lambda) \exp \left\{ -\frac{1}{2} \sum_{i=1}^n \left[x_i \kappa_n^2 \eta_i^{\perp 2} + \frac{m_n^2}{x_i \kappa_n^2} \right] \right\}. \quad (4)$$

The constant $\tilde{A}_n(\lambda)$ determines the probability amplitude for finding in a hadron the n constituents corresponding to scale λ . The $\kappa_n = \kappa_n(\lambda)$ is a parameter of the n -particle wave function. The mass parameter m_n depends on the Fock sector because self-interactions of constituents vary from sector to sector. It will be demonstrated below that the Brodsky-de Téramond holography can be understood in terms of this model.

The Fourier transforms of our model wave functions to momentum variables are

$$\psi_p^{(n)}(\mathbf{p}^\perp, \mathbf{x}; \lambda) = 2(2\pi)^3 \delta\left(\sum_{i=1}^n x_i - 1\right) \delta^{(2)}\left(\mathbf{p}^\perp - \sum_{i=1}^n \mathbf{p}_i^\perp\right) \psi^{(n)}(\mathbf{k}^\perp, \mathbf{x}; \lambda), \quad (5)$$

where

$$\psi^{(n)}(\mathbf{k}^\perp, \mathbf{x}; \lambda) = \frac{A_n(\lambda)}{\kappa_n^n} \exp \left\{ -\frac{1}{2 \kappa_n^2} \sum_{i=1}^n \left[\frac{k_i^{\perp 2} + m_n^2}{x_i} \right] \right\}. \quad (6)$$

The bold symbol $\mathbf{k}^\perp = (k_i^\perp)_{i=1, \dots, n}$ and $k_i^\perp = p_i^\perp - x_i \mathbf{p}^\perp$ is the transverse relative momentum of i th constituent with respect to the center of mass of all constituents in a sector. The wave function $\psi^{(n)}(\mathbf{k}^\perp, \mathbf{x}; \lambda)$ always appears in conjunction with the δ -functions such as in $\psi_p^{(n)}(\mathbf{p}^\perp, \mathbf{x}; \lambda)$ in Eq. (5). Therefore, its support is limited by the conditions

$$\sum_{i=1}^n x_i = 1 \quad \text{and} \quad \sum_{i=1}^n k_i^\perp = 0. \quad (7)$$

Thus, only $n - 1$ of the n three-dimensional arguments of $\psi^{(n)}(\mathbf{k}^\perp, \mathbf{x}; \lambda)$ are independent. The coefficient A_n in Eq. (6) is related to $\tilde{A}_n$ in Eq. (4) by the Fourier transform.

A model made of scalar quanta certainly does not appear useful for explaining true hadron observables. However, it is useful in seeking the rules of approximating theories that do include the necessary other types of quanta. The goal of the example is only to illustrate the idea of using the QFT analog of Ehrenfest theorem [10] to explain the correspondence between QFT and its holographic approximation [11] including the difficulty of dealing with a renormalized theory in the FF Fock space.

3 Ehrenfest Equation in QFT

The Ehrenfest theorem [10] connects classical-mechanics to quantum-mechanics by averaging the dynamics of a system set by its wave-function. The analogous procedure in QFT described below, averages the dynamics of an active constituent over all effective constituents in all Fock components of a hadron state. The effective constituent basis in the Fock space is defined using the RGPEP, at some arbitrary value of the scale parameter λ [11]. We now explain the Ehrenfest averaging in QFT using the example of a state described in Sec. 2.

We assume that there exists a function $\psi(k^\perp, x)$ that effectively describes the state defined in Eqs. (1) to (6). We call this function the Ehrenfest function. In contrast to the Fock-space wave functions which we average over, the Ehrenfest function depends only on three variables, two transverse momenta, $k^\perp = (k^1, k^2)$, and the $+$ momentum fraction, x . Since the Ehrenfest function enters into observable hadronic matrix elements with small momentum transfers through equations that do not depend on the RGPEP scale λ , it must also not depend on λ .

Thanks to the very simple structure of our model state, the averaging over spectators in every Fock sector is reduced to integrating over their invariant mass squared, $\mathcal{M}^2$, with the space density ρ_n , which is defined by

$$\rho_n(\mathcal{M}^2) = \int [\kappa^\perp, \chi] 2(2\pi)^3 \delta\left(\sum_{j=1}^{n-1} \chi_j - 1\right) \delta^{(2)}\left(\sum_{j=1}^{n-1} \kappa_j^\perp\right) \delta\left(\mathcal{M}^2 - \sum_{j=1}^{n-1} \frac{\kappa_j^{\perp 2} + m_n^2}{\chi_j}\right), \quad (8)$$

where $\kappa^\perp = (\kappa_i^\perp)_{i=1,\dots,n-1}$ and $\chi = (\chi_i)_{i=1,\dots,n-1}$ are the relative momenta in the system of $n-1$ spectators. Therefore, we can write the expectation value of $P^+ \hat{P}^- - P^{\perp 2}$ in a state of Eq. (1) as

$$M^2 = \left\langle \int_{k,x} \psi_{k,x}^{\dagger(n)} \left[\frac{k^{\perp 2}}{x(1-x)} + \frac{m_n^2}{x} + \frac{\mathcal{M}^2}{1-x} \right] \psi_{k,x}^{(n)} \right\rangle + \langle \text{connected interactions} \rangle. \quad (9)$$

The integration symbol $\int_{k,x}$ denotes the integration over $k^\perp$ and x weighted by inverse of $2(2\pi)^3 x(1-x)$. The symbol $\langle \rangle$ denotes integrating over $\mathcal{M}^2$ with the density ρ_n and summing over n . The functions

$$\psi_{k,x}^{(n)} = \frac{A_n}{\chi_n^n} \exp \left\{ -\frac{1}{2\chi_n^2} \left[\frac{k^{\perp 2}}{x(1-x)} + \frac{m_n^2}{x} + \frac{\mathcal{M}^2}{1-x} \right] \right\}, \quad (10)$$

result from collecting spectators into the core of invariant mass $\mathcal{M}^2$, which varies from $[(n-1)m_n]^2$ to infinity. This simplification is an important property of our symmetric Gaussian example. The mass parameter m_n results from inclusion of self-interactions that depend on the Fock sector number [11].

We recall that the motion of any system close to the quadratic minimum of potential energy is described by a Gaussian form and the lowest excited states arrange themselves according to a harmonic oscillator pattern. Therefore, even if we do not know the Gaussian widths χ_n and amplitudes A_n for all relevant values of n at any given λ , we can still trace the consequences of averaging various quantities in our example and derive the Ehrenfest formulae which gives insights for hadrons in QFT irrespective of the errors introduced by our minimal, Gaussian approximation.

The averaging in Eq. (9) yields the expectation values

$$\left\langle \int_{k,x} \chi_n^2 |\psi_{k,x}^{(n)}|^2 \right\rangle = \chi^2, \quad (11a)$$

$$\left\langle \int_{k,x} m_n^2 |\psi_{k,x}^{(n)}|^2 \right\rangle = m_{\text{active}}^2, \quad (11b)$$

$$\left\langle \int_{k,x} \mathcal{M}^2 |\psi_{k,x}^{(n)}|^2 \right\rangle = m_{\text{core}}^2. \quad (11c)$$

The quantity χ in Eq. (11a) is the half-width of the Ehrenfest function, whereas the quantities χ_n are the widths of the wave functions $\psi_{k,x}^{(n)}$. The width χ is an observable. It corresponds to the inverse of a hadron size [11].

The hadron size depends on all the Fock sectors, whose wave functions depend on the RGPEP scale λ , but as an observable measurable in terms of form factors it does not depend on λ . Hence, x must not depend on λ . The quantities m_{active}^2 and m_{core}^2 in Eqs. (11b) and (11c) are the expectation values for the active constituent and core masses squared, respectively. For example, for mesons built from light quarks in QCD, one may expect $m_{\text{active}} \sim m_{\text{core}} \sim \Lambda_{QCD}$ on the basis of the argument that the effective QCD at the RGPEP scale $\lambda \sim \Lambda_{QCD}$ resembles the quark-model picture of hadrons. By the same token, in light baryons, m_{core} is expected to resemble the mass of a diquark or perhaps be about twice larger than m_{active} in the quark model. Since m_{active} and m_{core} enter into λ -independent formulas for low-momentum-transfer hadronic observables, they cannot depend on λ . In addition to the mass terms, the expectation value of all connected interactions yields the effective interaction potential,

$$U_{\text{eff}} = \langle \text{connected interactions} \rangle, \quad (12)$$

between the active constituent and the core. We call this potential the Ehrenfest potential.

The averaged quantities obtained in Eqs. (11a–c) and (12), are used to write the expectation value in Eq. (9) as

$$\int_{k,x} \psi(k^\perp, x)^\dagger \left[\frac{k^\perp{}^2}{x(1-x)} + \frac{m_{\text{active}}^2}{x} + \frac{m_{\text{core}}^2}{1-x} + U_{\text{eff}} \right] \psi(k^\perp, x) = M^2. \quad (13)$$

Variation of this expectation value with respect to $\psi(k^\perp, x)$, keeping the norm of $\psi(k^\perp, x)$ fixed, yields

$$\left[\frac{k^\perp{}^2}{x(1-x)} + \frac{m_{\text{active}}^2}{x} + \frac{m_{\text{core}}^2}{1-x} + U_{\text{eff}} \right] \psi(k^\perp, x) = M^2 \psi(k^\perp, x), \quad (14)$$

which we call the Ehrenfest eigenvalue equation.

4 Ehrenfest Potential in AdS/QFT Duality

Our Gaussian model example produces eigenvalue equation for the LF Hamiltonian with a harmonic potential. Brodsky et al. [20] recently argued for a harmonic potential in LF-holography using the de Alfaro et al. [21] idea. The idea is based on the observation that a scale in quantum theory with conformal symmetry can still be introduced, but together with quadratic potential. Irrespective of this and other arguments, we state that our potential U_{eff} must satisfy two conditions. First, it should describe motion of the constituents close to a minimum of potential energy. This principle suggests a quadratic form for U_{eff} as a function of a properly defined distance between the active constituent and core. In principle there could be a higher even powers of the distance. Second, it must be defined as a function of the relative distance in such a way that the resulting mass spectrum agrees with the requirement of rotational symmetry, i.e. the multiplets of the spectrum must have multiplicities allowed by the symmetry.

In order to define the relative distance as a quantum-mechanical conjugated quantity to the relative momentum of active constituent and core, we observe that in the non-relativistic limit

$$x \simeq \frac{m_{\text{active}} + k^3}{m_{\text{active}} + m_{\text{core}}}, \quad \text{and} \quad k^3 \simeq m_{\text{core}} x - m_{\text{active}} (1-x). \quad (15)$$

Therefore, following [13], we define an exact expression for a three-dimensional relative momentum variable $\mathbf{k}$ by the formula

$$\mathbf{k} = (k_x, k_y, k_z) = \frac{(k^\perp, k^3)}{\sqrt{x(1-x)}}. \quad (16)$$

The Ehrenfest equation, Eq. (14), then reads

$$[\mathbf{k}^2 + (m_{\text{active}} + m_{\text{core}})^2 + U_{\text{eff}}] \psi(\mathbf{k}) = M^2 \psi(\mathbf{k}). \quad (17)$$

According to quantum mechanics, the potential U_{eff} around its minimum must have the form

$$U_{\text{eff}} = -\kappa^4 \left(\frac{\partial}{\partial \mathbf{k}} \right)^2 - \mathcal{B}, \quad (18)$$

where $\mathcal{B}$ is a free parameter. The normalized ground-state solution of Eq. (17) of the hadron charge distribution, is

$$\psi(k^\perp, x) \simeq \psi(\mathbf{k}) = \mathcal{N} e^{-\mathbf{k}^2/(2\kappa^2)}, \quad (19)$$

where $\mathcal{N}$ is a normalization factor. It is shown below that this solution matches the Brodsky–de Téramond wave function.

5 Connection Between the Ehrenfest Approximation and LF-Holography

The Ehrenfest picture can be connected with LF holography by comparing the three-dimensional Ehrenfest equation with a harmonic potential and the Brodsky–de Téramond two-dimensional eigenvalue equation with a transverse harmonic oscillator potential associated with a corresponding AdS dilaton warping [13]. Separating variables in Eq. (17), one obtains solutions in the form

$$\psi(\mathbf{k}) = \phi(k_x, k_y) H_{n_z} \left(\frac{k_z}{\kappa} \right) e^{-k_z^2/2\kappa^2}, \quad (20)$$

where H_{n_z} denotes a Hermite polynomial. Assuming that the transverse motion corresponds to the angular momentum projection on z -axis equal l_z , one arrives at

$$\left[-\left(\frac{\partial}{\partial \zeta} \right)^2 - \frac{1}{\zeta} \frac{\partial}{\partial \zeta} + \frac{1}{\zeta^2} l_z^2 + (2n_z + 1) \kappa^2 + \kappa^4 \zeta^2 - \mathcal{B} + (m_{\text{active}} + m_{\text{core}})^2 \right] \tilde{\phi}(\zeta) = M^2 \tilde{\phi}(\zeta). \quad (21)$$

The function $\tilde{\phi}(\zeta)$ is the radial factor in the Fourier transform of $\phi(k_x, k_y)$ and $\zeta = |\zeta^\perp|$ is the length of the Brodsky–de Téramond holography transverse position variable, identified in the Ehrenfest picture as

$$\zeta^\perp = \sqrt{x(1-x)} \left(r_{\text{active}}^\perp - r_{\text{core}}^\perp \right), \quad (22)$$

where the variables $r_{\text{active}}^\perp$ and $r_{\text{core}}^\perp$ denote the transverse positions of the active parton and the core at $x^+ = 0$. The variable x in the holographic factor $\sqrt{x(1-x)}$, is the LF hadron momentum-fraction carried by the active effective Ehrenfest parton. It can be also identified in the infinite momentum frame [22]. In terms of $\phi(\zeta) = \sqrt{\zeta} \tilde{\phi}(\zeta)$, Eq. (21) precisely matches the holography Eq. (11) in Ref. [9] after adjusting the additive constant $(m_{\text{active}} + m_{\text{core}})^2 - \mathcal{B}$.

Our model does not predict the value of the constant $\mathcal{B}$ for various hadrons, but can be equated with de Téramond et al. predictions [23]. Since the constant κ in the Ehrenfest U_{eff} may be thought to be related to the gluon condensate inside hadrons [13], one may expect that an explanation of the constant $\mathcal{B}$ in U_{eff} also requires understanding of the quark and gluon dynamics at distances comparable with the size of hadrons. It would be interesting to study the role of the Ehrenfest interpretation of partons in the context of searches for the origin of the proton spin [24].

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