Bernard L. G. Bakker · Chueng-Ryong Ji

Kinematical Issues in the GPD Formulation of DVCS

Received: 16 September 2011 / Accepted: 14 February 2012 / Published online: 13 March 2012
© The Author(s) 2012. This article is published with open access at Springerlink.com

Abstract The kinematics used in computing deeply virtual Compton scattering makes a significant difference in terms of the widely used reduced operators that define generalized parton distributions. We analyze this difference at tree-level.

1 Introduction

Generalized parton distributions are understood to embody new information beyond what one can learn from deep-inelastic scattering. Thus, it is important to analyze how GPDs can be extracted from the data. In the literature, some reference frame is considered “convenient” [1] or some condition is forced upon the external momenta [2]. This raises the question what are the “best” reference frames suited for the extraction of GPDs.

In a simpler situation, namely the determination of form factors, a frame where the transferred momentum q^μ has vanishing plus component, $q^+ = 0$, can be justified by appealing to the spectrum condition in LFD, which says that a (virtual) photon with $q^+ = 0$ cannot produce a massive particle–anti-particle pair. Consequently, such a photon would probe the pre-existing parton structure of a hadron, rather than induce new structures. This frame has been widely used.

However, we have shown explicitly [3] that in the $q^+ = 0$ frame where the virtual photon must have hard transverse momentum, the application of the GPD formulas tailored to a fully collinear kinematics [1], would give wrong results in the solvable case of tree-level DVCS, if one drops the contribution from the longitudinal polarization of the virtual photon, which is not justified in the $q^+ = 0$ frame. This motivated us to extend our analysis to include more general reference frames. In particular, the experimental setup at JLab does not allow for collinear kinematics [4,5] and, moreover, sets a limit on the Mandelstam variable t , namely $-t > |t_{\min}|$. This raises the question how important the higher-twist corrections of type t/q^2 are. Higher-twist effects are currently extensively discussed in the literature [6–11,15].

2 Original Formulation of DVCS in Terms of GPDs

We discuss here the hadronic part at tree level, where the GPDs are constants. We shall compare the exact DVCS amplitudes in leading order to the ones given in the original forms by Ji [1] and Radyushkin [2], understood to be valid in leading twist.

Presented by Ben Bakker at LIGHTCONE 2011, 23–27 May, 2011, Dallas.

B.L.G. Bakker (✉)
Department of Physics and Astrophysics, Vrije Universiteit, Amsterdam, The Netherlands
E-mail: b.l.g.bakker@vu.nl

C.-R. Ji
Department of Physics, North Carolina State University, Raleigh, NC 27695-8202, USA
E-mail: crji@ncsu.edu

First we introduce the kinematics of Ref. [1] where Sudakov vectors $n(\pm)$ are used (our notation)

$$\begin{aligned} n(+)^{\mu} &= (1, \mathbf{0}_{\perp}, 0), \quad n(-)^{\mu} = (0, \mathbf{0}_{\perp}, 1), \quad n(\pm)^2 = 0, \quad n(+)^{\mu} \cdot n(-)^{\mu} = 1, \\ \bar{P} &= \frac{P + P'}{2} = P^+ n(+) + \frac{\bar{M}^2}{2P^+} n(-), \quad q = -\xi P^+ n(+) + \frac{Q^2}{2\xi P^+} n(-), \\ \Delta &= P' - P = q - q' = -\xi P^+ n(+) + \xi \frac{\bar{M}^2}{2P^+} n(-) + \Delta_{\perp}, \end{aligned} \quad (1)$$

and the physical momenta

$$\begin{aligned} P &= (1 + \xi/2) P^+ n(+) + (1 - \xi/2) \frac{\bar{M}^2}{2P^+} n(-) - \frac{\Delta_{\perp}}{2}, \\ P' &= (1 - \xi/2) P^+ n(+) + (1 + \xi/2) \frac{\bar{M}^2}{2P^+} n(-) + \frac{\Delta_{\perp}}{2}, \quad q' = \left(\frac{Q^2}{2\xi P^+} - \xi \frac{\bar{M}^2}{2P^+} \right) n(-) - \Delta_{\perp}. \end{aligned} \quad (2)$$

Although $\Delta_{\perp}$ is introduced here, we note that $q'^2 = \Delta_{\perp}^2 = -\Delta_{\perp}^2 = 0$ and the kinematics in Ref. [1] is completely collinear as the final-state photon becomes real. Other invariants (with the understanding that $\Delta_{\perp} = 0$ for real photons in the final state) are expressed as

$$\begin{aligned} \bar{P}^2 &= \bar{M}^2, \quad P^2 = P'^2 = \frac{\Delta^2}{4} + \bar{M}^2 = M^2, \quad q^2 = -Q^2, \\ s &= \frac{2 - \xi}{2\xi} Q^2 + \frac{(2 - \xi)^2}{4} \bar{M}^2 - \frac{\Delta_{\perp}^2}{4}, \quad t = \Delta^2 = -\xi^2 \bar{M}^2 - \Delta_{\perp}^2, \\ u &= -\frac{2 + \xi}{2\xi} Q^2 + \frac{(2 + \xi)^2}{4} \bar{M}^2 - \frac{\Delta_{\perp}^2}{4}, \quad \xi = \frac{Q^2 - \xi^2 \bar{M}^2}{2\bar{P} \cdot q} \approx x_{\text{Bj}}. \end{aligned} \quad (3)$$

For $\Delta_{\perp} = 0$ and neglecting the mass of the hadrons compared to Q in DVCS, ξ is close to the invariant quantity x_{Bj} and $t = 0$ may also be taken in this collinear kinematics. Then a hadronic tensor is defined by

$$\begin{aligned} T^{\mu\nu}(P, q, \Delta) &= -\frac{1}{2} (n(+)^{\mu} n(-)^{\nu} + n(+)^{\nu} n(-)^{\mu} - g^{\mu\nu}) \int dx \left(\frac{1}{x - \xi/2 + i\epsilon} + \frac{1}{x + \xi/2 + i\epsilon} \right) \\ &\times \left[H(x, \Delta^2, \xi) \bar{U}(P') \not{n}(-) U(P) + E(x, \Delta^2, \xi) \bar{U}(P') \frac{i\sigma^{\alpha\beta} n(-)_{\alpha} \Delta_{\beta}}{2M} U(P) \right] \\ &- \frac{i}{2} \epsilon^{\mu\nu\alpha\beta} n(+)^{\alpha} n(-)^{\beta} \int dx \left(\frac{1}{x - \xi/2 + i\epsilon} - \frac{1}{x + \xi/2 + i\epsilon} \right) \\ &\times \left[\tilde{H}(x, \Delta^2, \xi) \bar{U}(P') \not{n}(-) \gamma_5 U(P) + \tilde{E}(x, \Delta^2, \xi) \frac{n \cdot \Delta}{2M} \bar{U}(P') \gamma_5 U(P) \right]. \end{aligned} \quad (4)$$

This equation is valid only in the special collinear reference frame defined in Ref. [1]. We have shown this [3] investigating the complete DVCS amplitude contracting the hadronic tensor with the polarization vectors $\epsilon^*(q')$ and $\epsilon(q)$.

Radyushkin [2] expresses his operator in terms of physical momenta P , Δ , and q' . The hard momentum q' is a Sudakov vector: $q'^2 = 0$. The skewness ζ is defined as $\zeta = x_{\text{Bj}} = Q^2/2P \cdot q$. The momenta are supposed to satisfy the relations $q = q' - \zeta P$, $P' = (1 - \zeta)P$.

This kinematics leads to the relation $t = \Delta^2 = (\zeta P)^2 = \zeta^2 M^2 > 0$, while DVCS requires $t \leq 0$. Thus, this kinematics becomes consistent with DVCS if one sets the nucleon mass M to zero, and thus $t \rightarrow 0$. In this limit Ji's ξ and Radyushkin's ζ are related by $\zeta = \frac{2\xi}{2+\xi}$, $\xi = \frac{2\zeta}{2-\zeta}$. If the mass of the target nucleon may be neglected in the kinematics, P and q' can also be considered Sudakov vectors, i.e., $P^{\mu}/P^+ = n^{\mu}(+)$ and

$q'^\mu/q'^- = P^+q'^\mu/P \cdot q' = n^\mu(-)$. The hadronic tensor is written by Radyushkin as follows (our notation)

$$\begin{aligned}
 T^{\mu\nu}(P, q, q') = & -\frac{1}{2P \cdot q'} \left(-g^{\mu\nu} + \frac{P^\mu q'^\nu + P^\nu q'^\mu}{P \cdot q'} \right) \int dx \left(\frac{1}{x - \zeta + i\epsilon} + \frac{1}{x - i\epsilon} \right) \\
 & \times \left[H(x, \eta, t) \bar{U}(P') \not{q}' U(P) + E(x, \zeta, t) \bar{U}(P') \frac{i\sigma^{\alpha\beta} q'_\alpha \Delta_\beta}{2M} U(P) \right] \\
 & - \epsilon^{\mu\nu\alpha\beta} \frac{P_\alpha q'_\beta}{P \cdot q'} \int dx \left(\frac{1}{x - \zeta + i\epsilon} - \frac{1}{x - i\epsilon} \right) \\
 & \times \left[\tilde{H}(x, \zeta, t) \bar{U}(P') \not{q}' \gamma_5 U(P) + \tilde{E}(x, \zeta, t) \frac{q' \cdot \Delta}{2M} \bar{U}(P') \gamma_5 U(P) \right].
 \end{aligned} \tag{5}$$

Despite the fact that the hadronic tensor is written in terms of physical momenta, this kinematics is accurate in leading twist only. In a frame where all physical momenta are aligned, i.e., the extreme collinear kinematics, the two formulations are equivalent. Here we are interested in the differences between the collinear kinematics and a realistic one, e.g., in JLab.

3 Tree-Level Calculation

We performed a tree-level calculation of DVCS, which is relevant for the hard-scattering part of the physical DVCS amplitude. It avoids computational complexity and issues of model building. Our results [3] are obtained for structureless, massless spin-1/2 hadrons and massless electrons. We expect that the differences found here will have consequences for realistic physical situations.

We use the overall plus-momentum scale P^+ . The hadron momenta are now called k and k' , which corresponds to the tree-level amplitude to be understood as the hard-scattering part of the DVCS amplitude. The hadron mass is set to 0. The following momenta are kept fixed

$$\begin{aligned}
 \ell^\mu &= \frac{1}{Q^4 - 4(\zeta P^+)^4} \left(4(\zeta P^+)^5, 2Q^3(\zeta P^+)^2, 0, \frac{Q^6}{2\zeta P^+} \right) \rightarrow \left(0, 0, 0, \frac{Q^2}{2\zeta P^2} \right) \text{ for } Q \rightarrow \infty, \\
 \ell'^\mu &= \frac{1}{Q^4 - 4(\zeta P^+)^4} \left(Q^4 \zeta P^+, 2Q^3(\zeta P^+)^2, 0, 2Q^2(\zeta P^+)^3 \right) \rightarrow (\zeta P^+, 0, 0, 0) \text{ for } Q \rightarrow \infty, \\
 k^\mu &= (\zeta P^+, 0, 0, 0).
 \end{aligned} \tag{6}$$

Three types of kinematics are used ($\mathbf{K}_1$, used earlier [12], is not discussed here).

$\mathbf{K}_2$ Kinematics, completely collinear

$$k'^\mu = ((x - \zeta)P^+, 0, 0, 0), \quad q^\mu = \left(-\zeta P^+, 0, 0, \frac{Q^2}{2\zeta P^+} \right), \quad q'^\mu = \left(0, 0, 0, \frac{Q^2}{2\zeta P^+} \right). \tag{7}$$

$\mathbf{K}_3$ Non-vanishing q^+ and q'^+ kinematics

$$k'^\mu = ((x - \zeta)P^+, 0, 0, 0), \quad q^\mu = \left(-\frac{\zeta}{2}P^+, \frac{Q}{\sqrt{2}}, 0, \frac{Q^2}{2\zeta P^+} \right), \quad q'^\mu = \left(\frac{\zeta}{2}P^+, \frac{Q}{\sqrt{2}}, 0, \frac{Q^2}{2\zeta P^+} \right). \tag{8}$$

The Mandelstam variables in these two kinematics are the same, in particular $t = 0$.

To see the effect of taking $t < 0$ we mimic the kinematics at JLab in kinematics $\mathbf{K}_4$, where the final hadron and final photon move off the z -axis

$$k'^\mu = \left((x - \zeta_{\text{eff}})P^+, \Delta_\perp, \frac{\Delta_\perp^2}{2(x - \zeta_{\text{eff}})P^+} \right), \quad q'^\mu = \left(\alpha \frac{\Delta_\perp^2}{Q^2} P^+, -\Delta_\perp, \frac{Q^2}{2\alpha P^+} \right), \tag{9}$$

while the initial hadron and virtual photon move along the z -axis, i.e., k^μ and q^μ are given in Eqs. 6 and 7. The quantity ζ_{eff} , determined by four-momentum conservation, is given by

$$\zeta_{\text{eff}} = \zeta + \alpha \frac{\Delta_\perp^2}{Q^2} \rightarrow \zeta \text{ for } Q \rightarrow \infty, \quad \alpha = \frac{x - \zeta}{2} \left(1 - \sqrt{1 - \frac{4\zeta}{x - \zeta} \frac{\Delta_\perp^2}{Q^2}} \right) \rightarrow 0 \text{ for } Q \rightarrow \infty. \tag{10}$$

The complete amplitude is the amplitude for the physical process $\ell + h \rightarrow \ell' + h' + \gamma'$

$$\mathcal{M} = \sum_h \mathcal{L}(\{\lambda', \lambda\}h) \frac{1}{q^2} \mathcal{H}(\{s', s\}\{h', h\}), \quad (11)$$

which splits into a leptonic and a hadronic part

$$\mathcal{L}(\{\lambda', \lambda\}h) = \bar{u}_{\text{LF}}(\ell'; \lambda') \not{\epsilon}^*(q; h) u_{\text{LF}}(\ell; \lambda), \quad \mathcal{H}(\{s', s\}\{h', h\}) = \bar{u}_{\text{LF}}(k'; s') (\mathcal{O}_s + \mathcal{O}_u) u_{\text{LF}}(k; s). \quad (12)$$

The operators occurring are, for massless hadrons

$$\mathcal{O}_s = \frac{\not{\epsilon}_{\text{LF}}^*(q'; h') (\not{k} + \not{q}) \not{\epsilon}_{\text{LF}}(q; h)}{(k + q)^2}, \quad \mathcal{O}_u = \frac{\not{\epsilon}_{\text{LF}}(q; h) (\not{k} - \not{q}') \not{\epsilon}_{\text{LF}}^*(q'; h')}{(k - q')^2}. \quad (13)$$

The polarization vectors are defined in Ref. [13] and the spinors are those given in Ref. [14]

4 Results in Three Kinematics

The leptonic amplitudes are displayed in Table 1 and the hadronic parts in Table 2.

The difference between $\mathbf{K}_2$ and $\mathbf{K}_3$ shows the difference of the contributions from the longitudinal virtual photon between the two kinematics. The difference in the hadronic amplitudes between $\mathbf{K}_2$ and $\mathbf{K}_3$ is also related to the definition of the helicity of the final-state photon, and the difference with $\mathbf{K}_4$ is due to the difference in t , being less than 0 in that kinematics.

Now we compare the results obtained from different formulations, namely Refs. [1], [2], with our solvable-model results. We shall show in our calculations that the terms missing in the other two are of order $\Delta_{\perp}^2/Q^2$. This confirms that the use of Eqs. 4 and 5 are limited to the lowest twist.

Rewriting the s - and u -channel hadronic amplitudes as

$$\bar{u}(k'; s') \mathcal{O}_s u(k; s) = \epsilon_{\mu}^*(q'; h') \epsilon_{\nu}(q; h) T_s^{\mu\nu}, \quad \bar{u}(k'; s') \mathcal{O}_u u(k; s) = \epsilon_{\mu}^*(q'; h') \epsilon_{\nu}(q; h) T_u^{\mu\nu}, \quad (14)$$

we find

$$T_s^{\mu\nu} = \frac{k_{\alpha} + q_{\alpha}}{s} \bar{u}(k'; s') \gamma^{\mu} \gamma^{\alpha} \gamma^{\nu} u(k; s), \quad T_u^{\mu\nu} = \frac{k_{\alpha} - q'_{\alpha}}{u} \bar{u}(k'; s') \gamma^{\nu} \gamma^{\alpha} \gamma^{\mu} u(k; s), \quad (15)$$

Table 1 Leptonic amplitudes in three kinematics

$\{\lambda', \lambda\}$	h	$\mathcal{L}(\{\lambda', \lambda\}h)$	$\mathbf{K}_2$	$\mathbf{K}_3$	$\mathbf{K}_4$
$\{\frac{1}{2}, \frac{1}{2}\}$	+1		0	$2Q$	0
$\{\frac{1}{2}, \frac{1}{2}\}$	0		0	$4iQ$	0
$\{\frac{1}{2}, \frac{1}{2}\}$	-1		$-2Q$	$-4Q$	$-2Q$

Table 2 Hadronic amplitudes in DVCS in three kinematics

$\{h', h\}$	$\{s', s\}$	$\mathcal{H}(\{h', h\}\{s', s\})$	$\mathbf{K}_2$	$\mathbf{K}_3$	$\mathbf{K}_4$
$\{+1, +1\}$	$\{\frac{1}{2}, \frac{1}{2}\}$	$2\sqrt{\frac{x-\zeta}{x}}$	$-2\sqrt{\frac{x-\zeta}{x-\zeta}}$	0	
$\{+1, +1\}$	$\{-\frac{1}{2}, -\frac{1}{2}\}$	$2\sqrt{\frac{x}{x-\zeta}}$	$-2\sqrt{\frac{x-\zeta}{x}}$	$-\frac{2\zeta^2}{\sqrt{x(x-\zeta)}(x-\zeta)} \frac{\Delta_{\perp}^2}{Q^2}$	
$\{-1, +1\}$	$\{\frac{1}{2}, \frac{1}{2}\}$	0	$4\sqrt{\frac{x-\zeta}{x}}$	$2\sqrt{\frac{x-\zeta}{x}} \left(1 - \frac{\zeta \Delta_{\perp}^2}{2(x-\zeta)Q^2}\right)$	
$\{-1, +1\}$	$\{-\frac{1}{2}, -\frac{1}{2}\}$	0	$4\sqrt{\frac{x}{x-\zeta}}$	$2\sqrt{\frac{x}{x-\zeta}} \left(1 + \frac{\zeta \Delta_{\perp}^2}{2(x-\zeta)Q^2}\right)$	
$\{+1, 0\}$	$\{\frac{1}{2}, \frac{1}{2}\}$	0	$-i4\sqrt{\frac{x}{x-\zeta}}$	0	
$\{+1, 0\}$	$\{-\frac{1}{2}, -\frac{1}{2}\}$	0	$-i4\sqrt{\frac{x-\zeta}{x}}$	$-i\frac{2\sqrt{2}\zeta}{\sqrt{x(x-\zeta)}} \frac{ \Delta_{\perp} }{Q}$	

Table 3 Complete DVCS amplitudes, $\sum_h \mathcal{L}(\{\lambda', \lambda\}, h) \frac{1}{q^2} \mathcal{H}(\{h', h\}\{s', s\})$ in three approaches, ours, Radyushkin, and Ji in $\mathbf{K}_4$. For massless hadrons and leptons $\lambda' = \lambda$ and $s' = s$

λ	h'	s	This work	AVR	XJ
$\frac{1}{2}$	1	$\frac{1}{2}$	$\frac{4}{Q} \sqrt{\frac{x}{x-\zeta}} \left(1 + \frac{\zeta}{2(x-\zeta)} \frac{\Delta_\perp^2}{Q^2} \right)$	$\frac{4}{Q} \sqrt{\frac{x}{x-\zeta}} \left(1 - \frac{\zeta}{2(x-\zeta)} \frac{\Delta_\perp^2}{Q^2} \right)$	0
$\frac{1}{2}$	1	$-\frac{1}{2}$	$\frac{4}{Q} \sqrt{\frac{x-\zeta}{x}} \left(1 - \frac{\zeta}{2(x-\zeta)} \frac{\Delta_\perp^2}{Q^2} \right)$	$\frac{4}{Q} \sqrt{\frac{x-\zeta}{x}} \left(1 + \frac{\zeta}{2(x-\zeta)} \frac{\Delta_\perp^2}{Q^2} \right)$	0
$\frac{1}{2}$	-1	$\frac{1}{2}$	$-\frac{4}{Q^3} \frac{\zeta^2}{\sqrt{x(x-\zeta)(x-\zeta)}} \frac{\Delta_\perp^2}{Q^2}$	0	$\frac{4}{Q} \sqrt{\frac{x}{x-\zeta}} \left(1 - \frac{\zeta}{2(x-\zeta)} \frac{\Delta_\perp^2}{Q^2} \right)$
$\frac{1}{2}$	-1	$-\frac{1}{2}$	0	0	$\frac{4}{Q} \sqrt{\frac{x-\zeta}{x}} \left(1 + \frac{\zeta}{2(x-\zeta)} \frac{\Delta_\perp^2}{Q^2} \right)$

respectively. Using the identity

$$\gamma^\mu \gamma^\alpha \gamma^\nu = g^{\mu\alpha} \gamma^\nu + g^{\alpha\nu} \gamma^\mu - g^{\mu\nu} \gamma^\alpha + i\epsilon^{\mu\alpha\nu\beta} \gamma_\beta \gamma_5, \quad (16)$$

one finds for T_s in leading order of Q

$$T_s^{\mu\nu} = \frac{q^-}{s} [\{n^\mu(-)n^\nu(+) + n^\nu(-)n^\mu(+) - g^{\mu\nu}\} \times \bar{u}(k'; s') \not{n}(-) u(k; s) - i\epsilon^{\mu\nu\alpha\beta} n_\alpha(-) n_\beta(+) \times \bar{u}(k'; s') \not{n}(-) \gamma_5 u(k; s)]. \quad (17)$$

A similar expression is found for T_u . This expression is equivalent to those given in Refs. [1], [2]. One fully recovers in this approximation the expressions given in Refs. [1,2] for the hadronic tensors after adding the s - and u -channel parts. Upon contracting them with the polarization vectors, we obtain full agreement in the hadronic amplitudes between $\mathbf{K}_2$ and $\mathbf{K}_4$ in the DVCS limit except for the swap between the amplitudes with $h' = +1$ and -1 . This swap will be explained below in our complete calculation. In kinematics $\mathbf{K}_3$ and $\mathbf{K}_4$ there is, however, a discrepancy. To understand this we expand the momenta in Sudakov vectors, but retain the remainder, denoted as *perpendicular* parts.

In our kinematics where q and k are aligned with the z -axis we have ($k^- = 0$ in the massless case)

$$\begin{aligned} q^\mu &= q^+ n^\mu(+) + q^- n^\mu(-), & q'^\mu &= q'^+ n^\mu(+) + q'^- n^\mu(-) + q'_\perp{}^\mu, \\ k^\mu &= k^+ n^\mu(+), & k'^\mu &= k'^+ n^\mu(+) + k'^- n^\mu(-) + k'_\perp{}^\mu \end{aligned} \quad (18)$$

In our more general kinematics $\mathbf{K}_4$ we thus find for T_s (see also Vanderhaegen et al. [6–11] where transverse-momentum components are included)

$$\begin{aligned} T_s^{\mu\nu} &= \frac{1}{s} [\{(k^+ + q^+) n^\mu(+) + q^- n^\mu(-) + q_\perp{}^\mu\} n^\nu(+) \\ &\quad + \{(k^+ + q^+) n^\nu(+) + q^- n^\nu(-) + q_\perp{}^\nu\} n^\mu(+) - g^{\mu\nu} q^- \times \bar{u}(k'; s') \not{n}(-) u(k; s) \\ &\quad - i\epsilon^{\mu\nu\alpha\beta} \{(k^+ + q^+) n_\alpha(+) + q^- n_\alpha(-) + q_{\perp\alpha}\} n_\beta(+) \times \bar{u}(k'; s') \not{n}(-) \gamma_5 u(k; s)]. \end{aligned} \quad (19)$$

By retaining only the terms proportional to the highest power in Q , namely those proportional to q^- , we obtain the approximate tensor shown in Eq. 17. However, the amplitudes being obtained by contraction with the polarization vectors are sensitive to the neglected parts as we show in Table 3, where we give the results for the complete amplitudes in kinematics $\mathbf{K}_4$ using the formalisms of Ref. [1] (XJ), Ref. [2] (AVR), and our exact calculation up to the order of $\Delta_\perp^2/Q^2$. The differences are summarized in the last Section. Owing to the definition of the LF helicity, it is opposite to the IF helicity if the momentum of the photon in the final state is pointing strictly in the $-z$ -direction. This *swap* explains the differences among the results in kinematics $\mathbf{K}_2$, $\mathbf{K}_3$, and $\mathbf{K}_4$ at $t = 0$ displayed in Table 4, where we summarize our results for the complete amplitudes. The sign $\pm$ in this Table denotes the helicity of the emitted photon.

Table 4 Comparison of the three approaches in three kinematics

K_2		K_3		$K_4 (t = 0)$
BC2($\pm$)	=	BC3($\mp$)	=	BC4($\mp$)
AVR2($\pm$)	=	AVR3($\mp$)	=	AVR4($\mp$)
XJ2($\pm$)	=	XJ3($\pm$)	=	XJ4($\pm$)

5 Summary and Conclusions

By applying the GPD formulation to an exactly solvable model, namely DVCS at tree level, we have been able to pinpoint the differences between the lowest-twist formulations of Refs. [1], [2] and the exact results.

1. In a kinematics where the hadrons and the photons are aligned (fully collinear kinematics K_2), the three approaches give identical results in the DVCS limit $Q \rightarrow \infty$ except for the swap between the amplitudes with $h' = +1$ and -1 . In kinematics K_4 we get agreement if we take there the limit $t \rightarrow 0$.
2. The effect of $\Delta_\perp$ is power suppressed as $\Delta_\perp^2/Q^2$. The amplitude with $\lambda = 1/2$, $h' = -1$, and $s = 1/2$ does not vanish to this order of $\Delta_\perp^2/Q^2$, while the original leading-twist formulas cannot generate this non-zero amplitude.
3. The formulation of Ref. [1] can be used if the difference between light-front and instant-form helicities are taken into account.
4. Since both lowest-twist approximations Refs. [1], [2] are accurate to zeroth order in $\Delta_\perp^2/Q^2$, the lowest-twist formulae can be used to test the sum rules that relate the form factors to the GPDs only at $t = 0$.

Having said this, we note that including the transverse momentum component in the kinematic tensor part of $T^{\mu\nu}$ does not completely cover the higher twist effects. The inclusion of higher twist effects [6–11, 15] is at the current forefront of research and it may well go beyond the handbag approximation.

We caution against using the lowest-twist formulas in the analysis of experimental data in situations where the net transverse-momentum transfer to the target is not small compared to Q . In this respect, it may be useful to note that e.g. in JLab the values of $\Delta_\perp^2/Q^2$ are larger than 5–10%.

Open Access This article is distributed under the terms of the Creative Commons Attribution License which permits any use, distribution, and reproduction in any medium, provided the original author(s) and the source are credited.

References

1. Ji, X.: Gauge-invariant decomposition of nucleon spin. Phys. Rev. Lett **78**, 610 (1997)
2. Radyushkin, A.V.: Non-forward parton distributions. Phys. Rev. D **56**, 5524 (1997)
3. Bakker, B.L.G., Ji, C.-R.: Spin filter in DVCS amplitudes. Phys. Rev. D **83**, 091502(R) (2011)
4. Bertin, P., et al., Deeply virtual Compton scattering on the neutron, Jefferson Lab. Proposal PR03-106
5. Mazouz, M., et al., Measurement of the deeply virtual Compton scattering cross-section off the neutron, Jefferson Lab. Proposal PR-08-025.
6. Vanderhaegen, M., Guichon, P.A.M., Guidal, M.: Hard electroproduction of photons and mesons on the nucleon. Phys. Rev. Lett **80**, 5064 (1998)
7. Guichon, P.A.M., Vanderhaegen, M.: Virtual Compton scattering off the nucleon. Prog. Part. Nucl. Phys. **41**, 125 (1998)
8. Anikin, I.V., Pire, B., Teryaev, O.V.: Gauge invariance of the deeply virtual Compton scattering amplitude. Phys. Rev. D **62**, 071501 (2000)
9. Kivel, N., Polyakov, M.V.: DVCS on the neutron to the twist-3 accuracy. Nucl. Phys. B **600**, 334 (2001)
10. Radyushkin, A.V., Weiss, C.: DVCS amplitude at tree-level: transversality, twist-3, and factorization. Phys. Rev. D **63**, 114012 (2001)
11. Radyushkin, A.V., Belitsky, A.V.: Unraveling hadron structure with generalized parton distributions. Phys. Rept. **418**, 1 (2005)
12. Bakker, B.L.G., Ji, C.-R.: GPD extraction from DVCS in LFD. Nucl. Phys. B (Proc. Suppl.) **199**, 16 (2010)
13. Bakker, B.L.G., Choi, H.-M., Ji, C.-R.: The vector-meson form-factor analysis in light-front dynamics. Phys. Rev. D **65**, 116001 (2002)
14. Kogut, J.B., Soper, D.E.: Quantum electrodynamics in the infinite-momentum frame. Phys. Rev. D **1**, 2901 (1970)
15. Braun, A. M., Manashov, A.N., Kinematic power corrections in off-forward hard reactions. ArXiv:1108.2394