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Hypothesis of Quark Binding by Condensation of Gluons in Hadrons

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Abstract Hypothesis of quark binding through condensation of gluons inside hadrons is formulated in the context of a renormalization group procedure for effective particles (RGPEP) in the light-front (LF) Hamiltonian approach to QCD. At the momentum scales of relative motion of hadronic constituents that are comparable with Λ_{QCD} , the hypothetical boost-invariant constituent dynamics is identified using gauge symmetry. The resulting picture of mesons and baryons closely resembles constituent quark models with harmonic oscillator potentials, shares some features of AdS/QCD, and can be systematically studied using RGPEP in QCD.

1 Canonical Approach to QCD

The canonical plan for solving QCD is to start from the corresponding classical gauge-invariant Lagrangian density, $\mathcal{L}_{can}$, derive from it a candidate for a Hamiltonian density, $\mathcal{H}_{can}$, and evaluate the Hamiltonian $H_{can} = \int d^3x \mathcal{H}_{can}$, in which the quark and gluon fields are quantized. Our discussion of the quantum theory begins with the standard form of Hamiltonian dynamics, called the instant form (IF), but our goal is to address the front form (FF) of the theory [1].

The canonical operator H_{can} acts in the Fock space of states $|\psi\rangle$ that are built from a vacuum by the operators that create quarks and gluons. Then, a solution to the eigenvalue problem

$$H_{can}|\psi\rangle = E|\psi\rangle, \quad (1)$$

with energy of the form $E = \sqrt{M_h^2 + \mathbf{P}^2}$, where M_h is a mass, should represent a hadron h moving with momentum $\mathbf{P}$. Time evolution of all states would be described using operators of the form $U(t_2, t_1) = e^{-iH_{can}(t_2-t_1)}$, etc. The ultimate goal of such plan for QCD is to achieve a quality of the wave function representation of hadrons that matches, and in the future hopefully even exceeds the quality of today's QED representation of atoms and their chemistry.

2 Time-Honored Problems of Canonical Approach

The canonical approach encounters divergence problems that are known for a long time to require a logical resolution [2]. A hint of the resolution was discovered also a long time ago [3] through the concept of a renormalization group (RG) procedure [4]. The idea is that one can *calculate* the counterterms required by

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the adopted regularization. This is done in a sequence of steps that eventually produce a manageable effective theory. To begin with, one has to replace H_{can} [5,6] with a regulated operator, H_{can}^Δ , where Δ stands for an extreme cutoff parameter. The counterterms, CT^Δ , that need to be included in the initial Hamiltonian, $H = H_{can}^\Delta + CT^\Delta$, are obtained from the condition that the matrix elements of the desired effective Hamiltonian do not depend on the regularization in H . The calculation of counterterms is based on evaluation of a whole family of effective Hamiltonians, H_λ labeled by the RG parameter λ , which can be chosen to have the dimension of momentum. This parameter plays the role of a sliding momentum cutoff.

The question that so far has no satisfactory resolution in canonical approaches to QCD is: What precisely is the operator H_λ in which λ is so small that the eigenvalue problem for H_λ can be solved using computers? One well-known reason for the difficulty is asymptotic freedom [7,8]. It implies that the effective coupling constant g_λ in H_λ increases according to the rule $g_\lambda \sim 1/\ln(\lambda/\Lambda_{QCD})$ when λ decreases (for the case of H_λ in LF QCD, see [9]), and becomes too large too soon for a perturbative procedure to produce H_λ with λ sufficiently small for a reliable computation of quark and gluon wave functions of hadrons.

The chief difficulty of the IF of dynamics, however, is that we do not know the ground state of QCD. The canonical IF interaction Hamiltonians are able to create virtual particles from empty space and as time flows they generate this way an infinitely complex state that so far nobody can describe [2]. The ground state, called vacuum, should be invariant with respect to the Lorentz transformations. But such transformations change momenta by arbitrary amounts and a theory with a finite cutoff on momentum cannot have a Lorentz invariant ground state. Nevertheless, a covariant perturbation theory in QCD allows for introduction of non-trivial parameters that can be interpreted as vacuum expectation values of operators. These parameters are related to the spectrum of hadrons through dispersion relations. This is how the quark and gluon vacuum condensates are introduced in the QCD sum rules [10,11].

In contrast to the case of the IF of dynamics, it is well-known that the FF of Hamiltonian dynamics necessarily leads to a trivial vacuum state, apparently allowing one to make progress without getting stuck in the hard vacuum problem right away. This fact motivated efforts to set up a similarity renormalization group (SRG) procedure for Hamiltonians [12,13] and apply it to LF QCD [14]. The effects that in the IF are associated with the unknown state of the vacuum, were suggested in Ref. [14] to be contained in the new terms in H , and thus also in H_λ , that a precise SRG procedure could identify. Through later efforts, summarized in Ref. [15], it was found that there exists an additional possibility for the vacuum-like terms to emerge in LF QCD in a systematic calculation. The additional possibility is the subject of this article.

3 Renormalization Group Procedure for Effective Particles

In the renormalization group procedure for effective particles (RGPEP) that is used in Ref. [15] to suggest a new locus for vacuum-like condensate effects in LF QCD, the key element of reasoning is a unitary connection between creation and annihilation operators for bare, canonical quarks and gluons, and the operators for effective particles,

$$a_s = U_s a_0 U_s^\dagger. \quad (2)$$

The parameter $s = 1/\lambda$ has interpretation of the size of the effective particles with respect to strong interactions. The parameter λ describes the momentum width of vertex form factors in interaction terms, and s , as its inverse, corresponds to the non-local interaction vertex range in space. Thus, the subscript 0 refers to the point-like particles, with only local interactions, while $s > 0$ implies non-local interactions [16]. Correspondingly, the effective quantum fields in space are constructed according to the well-known Fourier superposition rule [15]

$$\psi_s(x) = \int [p] a_{sp} e^{-ipx}. \quad (3)$$

As a result, $\psi_0(x)$ corresponds to canonical fields, and $\psi_s(x)$ to fields of effective quanta of size s . By writing $\psi_s(x) = \psi(x, s)$, one can realize that the scale-dependent effective theories form together a single 5-dimensional theory, the 5th dimension being the size of effective particles. It is natural to expect that this size is dynamically limited from above by $s_{QCD} \sim 1/\Lambda_{QCD}$ and that s_{QCD} corresponds to the depth of bulk penetration by matter fields in AdS/QCD models. We shall comment on this issue near the end of the article.

The transformation U_s in Eq. (2) is constructed in such a way (see [15] and references therein for computational details) that the Hamiltonian is not changed, $H_s(a_s) = H(a_0)$. But the Hamiltonians differ in their

structure: $H_s(a_s)$ is a combination of products of operators a_s with coefficients c_s that are different from the coefficients c_0 of corresponding products of operators a_0 in $H(a_0)$. RGPEP provides differential (or algebraic) equations that produce expressions for the coefficients c_s . From the equality

$$H_s(a_0) = U_s^\dagger H(a_0) U_s, \quad (4)$$

and the condition $U_0 = 1$, one obtains

$$\frac{d}{ds^4} H_s(a_0) = [G_s, H_s(a_0)], \quad (5)$$

with the generator

$$G_s = -U_s^\dagger \frac{d}{ds^4} U_s \quad (6)$$

and initial condition $H_0(a_0) = H(a_0) = H_{can}^\Delta + CT^\Delta$. The use of s^4 is due to dimensional reasons since G_s is designed to have dimension of mass to power 4. The non-perturbative, boost invariant generator G_s for RGPEP is given in Eq. (C.1) of Ref. [15]. It is expressed in terms of the operators a_0 in the form of a commutator,

$$G_s = [H_{free}, H_s^+], \quad (7)$$

where H_{free} denotes the part of $H(a_0)$ that involves only products of the form $a_0^\dagger a_0$ (one particle operators). H_s^+ is the remaining part of the Hamiltonian. The superscript $+$ indicates that the coefficients c_s in each and every term in H_s^+ are multiplied by the square of the total $+$ momentum of the particles that participate in the interaction described by a given term (see Eqs. (C.2) and (C.3) in [15]). As a result, when s increases, one obtains from Eq. (5) the effective interactions that are increasingly tempered by the vertex form factors that limit the changes of invariant masses of interacting effective particles. For example, solving Eq. (5) in a lowest order in powers of the interaction strength for any interaction term V , one obtains

$$\langle m(s) | V_s(a_s) | n(s) \rangle = e^{-s^4(\mathcal{M}_{Im}^2 - \mathcal{M}_{In}^2)^2} \langle m(s) | V_0(a_s) | n(s) \rangle, \quad (8)$$

where $|m(s)\rangle$ and $|n(s)\rangle$ denote arbitrary states in the Fock space built using operators a_s and $\mathcal{M}_{Im}$ and $\mathcal{M}_{In}$ denote the total invariant masses of only these subsets of effective particles in the corresponding states that are directly involved in the matrix element of the interaction V . The RGPEP vertex form factors suggest that the effective LF Fock space description of hadronic states may actually converge. This expectation needs to be verified by explicit calculations, which is a serious challenge.

Apparently similar to Eq. (5), beautiful flow equations have been developed by Wegner [17, 18]. The five main ways RGPEP differs are: (1) The transformation of a Hamiltonian in RGPEP is limited to transformations of creation and annihilation operators, which is a narrower class of transformations than rotating matrices, since matrices that result from evaluating matrix elements of linear combinations of products of rotated creation and annihilation operators form only a subset in the set of all matrices of interest in quantum mechanics; (2) Eq. (5) does not require specification of the full diagonal matrix elements of the evolving Hamiltonian, as Wegner's equation does, and H_{free} does not have to be just a part of H ; (3) The Hamiltonian as an operator is not altered at all by Eq. (5) since the coefficients c_s evolve in a way that is compensated by the evolution of operators a_s , but the RGPEP derivation of counterterms alters the initial condition at $s = 0$, a feature that Wegner's equation does not include, since it treats the initial Hamiltonian as given (the calculation of H is one of two generic goals of RGPEP, as it is in the SRG [12, 13], the other goal being the evaluation of corresponding effective Hamiltonians with s sufficiently large so that they can be used in numerical computations, and the latter goal is shared with Wegner's flows); (4) The RGPEP operator calculus renders coefficients c_s in the Hamiltonians $H_s(a_s)$ that can be applied to arbitrary states in the Fock space, instead of only to a specified set of states that is used in defining the Wegner matrix equations; (5) The generator G_s in Eq. (7) is constructed to preserve the 7 kinematical symmetries of the FF of Hamiltonian dynamics. The feature (5) is essential for application of RGPEP to QCD since it allows one to kinematically connect the picture of a hadron in its rest frame with the picture of the same hadron in the infinite momentum frame. This is a prerequisite for any formulation of QCD that aims at simultaneously explaining the constituent quark model classification of hadrons in the particle data tables and the parton distribution functions measured in deep inelastic scattering processes as well as other high-energy hadronic properties that manifest themselves in collisions of fast moving hadrons.

Fig. 1 Visualization of the idea that a three-quark configuration in a proton evolves with the RGPEP scale parameter s : **a** the picture at s much smaller than the scale s_c that corresponds to the constituent model, **b** s somewhat smaller than s_c , and **c** s comparable with s_c . The key consequence of the RGPEP is that the slow effective quarks must be large. For $s \sim 1/\Lambda_{QCD}$, they are as large as the proton itself. In the case **c**, they overlap a lot and balance color to zero inside a large part of the proton volume

4 Scale-Dependent Constituent Picture and the Gluon Condensate in Hadrons

Operators $W_{s_2 s_1} = U_{s_2} U_{s_1}^\dagger$ transform a hadron state $|\psi\rangle$ constructed in terms of quarks and gluons of size s_1 into the same state but constructed in terms of quarks and gluons of size s_2 . The effective quark mass depends on the RGPEP scale parameter s and, in the non-perturbative solutions of Eqs. (5) and (7), may be expected to evolve from a current quark mass at small s to a constituent quark mass for s of the order of s_c . Such increase is plausible as an extrapolation of the perturbative results [19] and perhaps can also be related to the results obtained in the non-perturbative calculations based on the Dyson–Schwinger equations [20]. Since the operators U_s depend on interactions, they change the number of virtual particles in states they act on. Consequently, $W_{s_2 s_1}$ also changes the number of particles. Thus, even if one assumes that for $s_1 \sim s_c = 1/\Lambda_{QCD}$ a proton can be represented as built from just three large constituent quarks, Fig. 1c, a change of scale from $s \sim s_c$ to $s < s_c$ is necessarily associated with creation of additional virtual particles whose presence is indicated in Figs. 1a and b by the large circle. Therefore, while the eigenvalue problem of $H_{s_c}(a_{s_c})$ for the proton state may take the form of a Schrödinger equation for three constituent quarks in a potential well, the eigenvalue problem of $H_s(a_s)$ with $s < s_c$ for the same proton state must take the form of a Schrödinger equation for three smaller quarks that are accompanied by additional virtual gluons (we ignore in this discussion additional quark-anti-quark pairs). These gluons are condensed only in the volume of the proton. The expectation value of the gluon field-strength operator squared in this cloud of gluons is described below as the parameter that plays the role of a gluon condensate parameter in the eigenvalue problems of effective LF Hamiltonians derived in QCD using RGPEP.

The title hypothesis of this article states that, although the FF vacuum is trivial, the hadronic gluon content is not, and it is the latter that provides the FF dynamical effects which are otherwise associated with the vacuum in the IF of dynamics. The remaining part of this article briefly describes results that support the title hypothesis.

5 Gauge Symmetry and the Effective Dynamics for Quarks of Size $s \lesssim s_c$

The eigenvalue problems for mesons (M) and baryons (B) expressed in terms of quarks and gluons from Fig. 1b, based on the relations $W_{s s_c} |12\rangle_{s_c} = |12G\rangle_s$ and $W_{s s_c} |123\rangle_{s_c} = |123G\rangle_s$, determine wave functions $\psi_s(12G)$ and $\psi_s(123G)$ in the corresponding states,

$$|M\rangle_s = \sum_{12G} \psi_s(12G) |12G\rangle_s, \quad |B\rangle_s = \sum_{123G} \psi_s(123G) |123G\rangle_s, \quad (9)$$

where the component G represents the gluons condensed in a hadron. The operator $G^\dagger$ that creates the gluon component of size of a hadron from the LF vacuum is approximated by a creation operator for a scalar particle. The effective eigenvalue problems for mesons and baryons, $H_s|h\rangle_s = E|h\rangle_s$, are projected on the basis states

$|12G\rangle_s$ and $|123G\rangle_s$, respectively, in order to obtain the wave functions $\psi_s(12G)$ and $\psi_s(123G)$. The basis states are constructed to include the color-transport factors between i th quark and a geometrical center, $\underline{x}$, of quarks in a hadron, $T_i = \exp -ig \int_{\underline{x}}^{x_i} dx_\mu A^\mu$, and the gluon field operator A is approximated using the Schwinger gauge by $A^\mu(x) = \frac{1}{2} (x - x_G)_\nu G^{\nu\mu}$, where x_G denotes the position of the gluon body created by $G^\dagger$ in the hadron and $G^{\mu\nu}$ is the gluon field strength operator at this point (see [15,21]).

Currently, in the absence of precise numerical information about the true low-energy effective Hamiltonian for LF QCD, local gauge symmetry is used to establish the structure of $H_s(a_s)$ for $s \lesssim s_c$ as follows [15]. Center-of-mass motion of an eigenstate of the LF Hamiltonian separates out completely from the eigenvalue problem and the equation one is left with is for an operator whose eigenvalue is the mass squared of a hadron. The operator itself is a sum of a free mass squared of constituents, denoted here by $\mathcal{M}^2$, plus the interactions which we do not know, say V . But we know that in the rest frame of constituent quarks their invariant mass squared is the square of the sum of their energies. These energies can be approximated by making a non-relativistic (NR) expansion, $E_p = m + p^2/(2m)$, since the RGPEP vertex form factors for $s \lesssim s_c$ prevent the interactions from accelerating constituent quarks to large relative speeds. Thus, one can sum the individual quark energies and square the sum, neglecting terms smaller than p^2 . The result is a simple quadratic expression in relative momenta of quarks. This result is easy to obtain for arbitrary masses of individual quarks, so that one knows the right coefficients with which the NR momenta squared enter the invariant mass squared. By comparison of this result with the exact expression for the invariant mass squared in the FF of dynamics, which is quadratic in the transverse relative momenta but a more complicated function of longitudinal momentum fractions carried by quarks, one learns how to write the latter in terms of the three-momenta known in the NR theory using LF variables [15]. Now the crux is that we also know how gauge symmetry dictates interaction through the minimal coupling in the NR theory: $\mathbf{p} \rightarrow \mathbf{p} - g\mathbf{A}$. So, by analogy, we also know how to estimate the effects of the minimal coupling in the LF mass squared. Namely, we trace the consequences of the minimal coupling in the NR theory and introduce the corresponding effects in the LF theory. As a result, we obtain a free invariant mass squared of quarks plus interaction terms V induced by expectation values of the gluon field in the gluon component G . Our result is the gauge symmetry candidate for effective $H_s(a_s)$.

In short, the NR reasoning proceeds along the lines of Ref. [21], except that on the basis of RGPEP on the LF one considers expectation values of $\mathbf{A}^2$ in the gluonic component of a hadron, $|G\rangle = G^\dagger|0\rangle$, rather than in the omnipresent vacuum $|\Omega\rangle$. After inclusion of the color transport factors, in a crude Abelian mean-field approximation, the expectation values of the type $\langle G|g_s^2 \mathbf{A}^2|G\rangle$, with $\mathbf{A} = \mathbf{B} \times \mathbf{r}/2$ and $\mathbf{B}$ being the magnetic part of $G^{\mu\nu}$, render harmonic potentials of the form $\varphi_G^2 \mathbf{r}^2$ where $\varphi_G^2 = \langle G|(\alpha_s/\pi) G^{\mu\nu c} G_{\mu\nu}^c|G\rangle / \langle G|G\rangle$ and $\mathbf{r}$ denotes the relative position of the quarks (only their relative position appears because of gauge symmetry).

By definition, in the momentum representation, the relative distances are defined as gradients with respect to the relative momenta. But we already know how to identify these momenta through the NR approximation that holds in the smallest mass eigenstates of RGPEP Hamiltonians with large s . Namely, in the case of mesons built from a quark of momentum p_1 , anti-quark of momentum p_2 (both having the same constituent quark mass m) and the glue component G , using notation $P = p_1 + p_2$, $x = p_1^+/P^+$, $p_1^\perp = xP^\perp + \kappa^\perp$, $p_2^\perp = (1-x)P^\perp - \kappa^\perp$, one has [15]

$$k^\perp = \frac{\kappa^\perp}{2\sqrt{x(1-x)}}, \quad k^z = \frac{2x-1}{2\sqrt{x(1-x)}} m, \quad (10)$$

and the effective quark dynamics in the gluon condensate inside a meson takes the form

$$\mathcal{M}_{q\bar{q}}^2 = 4m^2 + 4 \left[\mathbf{k}^2 + \frac{1}{2} m^2 \left(\frac{\pi\varphi_G}{3m} \right)^2 \frac{1}{2} \left(i \frac{\partial}{\partial \mathbf{k}} \right)^2 \right]. \quad (11)$$

For baryons one obtains (see [15] for definitions of momenta $\mathbf{K}$ and $\mathbf{Q}$)

$$\mathcal{M}_{3q}^2 = 9m^2 + 6\mathbf{K}^2 + \frac{9}{2}\mathbf{Q}^2 + 3m^2 \left(\frac{\pi\varphi_G}{3m} \right)^2 \frac{5}{8} \left[\frac{1}{2} \left(i \frac{\partial}{\partial \mathbf{K}} \right)^2 + \frac{2}{3} \left(i \frac{\partial}{\partial \mathbf{Q}} \right)^2 \right]. \quad (12)$$

The corresponding harmonic oscillator frequencies $\omega_M = \pi\varphi_G/(3m)$ and $\omega_B = \sqrt{5/8} \omega_M$ match phenomenologically accurate constituent quark models (e.g., see [22–24]) provided that one numerically identifies φ_G^2 with the vacuum gluon condensate parameter $\langle \Omega | (\alpha_s/\pi) G^{\mu\nu c} G_{\mu\nu}^c | \Omega \rangle / \langle \Omega | \Omega \rangle$ fitted to the spectrum of hadron masses in the QCD sum rules [10,11]. We reinterpret the vacuum condensate parameter as corresponding to

the gluons condensing only inside hadrons. The ground-state eigenfunctions of operators in Eqs. (11) and (12) are exponentials of the invariant mass squared of the effective quarks,

$$\psi_n = N \exp \left\{ -\frac{1}{2nm\omega} \left[\left(\sum_{i=1}^n p_i \right)^2 - (nm)^2 \right] \right\}, \quad (13)$$

where $n = 2$ for mesons and $n = 3$ for baryons, with $\omega = \omega_M$ and $\omega = \omega_B$, respectively.

6 Observables

Since the quarks are accompanied by the gluons, i.e., they move with respect to the gluon body G , the electro-weak hadron form factors are obtained in the form of the Fermi motion-smeared form factors for the effective quarks alone. However, the smearing is a small effect, so that the resulting form factors closely resemble results obtained in quark models. At the same time, RGPEP appears to provide the right tools for understanding a transition between the asymptotic counting-rules for hard scattering processes and the soft, non-perturbative effects in low-momentum transfer processes. Similar comments apply in the case of structure functions which depend on the Bjorken x and momentum transfer Q . In the RGPEP, the transformation $W_{s_Q s_c}$ provides a connection between the current quarks of a small size s_Q , i.e., the ones that are capable of a sudden absorption or emission of a hard-photon, and the effective constituent quarks of size $s_c \sim 1/\Lambda_{QCD}$. This transformation is expected to describe the evolution of parton distributions, universally in Q and x since the size parameter controls dependence on the invariant masses that are functions of Q and x simultaneously. Therefore, one may expect the same RGPEP tools to help shed some light on the gluon saturation mechanism in QCD as seen in the inelastic, inclusive or semi-inclusive processes.

It is also worth observing that the momentum variable $k^\perp = \kappa^\perp / \sqrt{x(1-x)}$ that Brodsky and de Teramond discovered in their LF holographic AdS/QCD picture of hadrons [25–27], appears to match the one dictated here by gauge symmetry and the condensation of gluons inside hadrons. This matching warrants further study, see Sect. 6.4 in [15] and Ref. [28]. We observe here that: (1) The AdS 5th dimension appears to correspond to the quark size s in RGPEP; (2) G -induced oscillator potentials appear to correspond to the soft-wall (SW) models [29]; (3) the resulting identification $\kappa_{SW}^2 = 2m\omega_M = (2\pi/3)\varphi_G$ suggests that the soft wall in the SW models results from the gluon condensation in hadrons; (4) AdS/QCD SW phenomenology result $\kappa_M/\kappa_B \sim 1.15 \pm 0.5$ matches our result for the same quantity, $(8/5)^{1/4} \sim 1.125$. These results support the idea that the condensates typically associated with the vacuum state are actually associated with the hadronic interior [30]. Since we discuss here only the gluon condensate, it should be mentioned that the quark condensate has also been suggested in the literature to be associated only with the hadronic interior [31,32].

The last comment we wish to make here is that the hadron content-induced harmonic oscillator potential between quarks at large RGPEP size s implies LF eigenvalues $M^2 \sim r^2$ for states where r is large. This quadratic behavior of M^2 implies $M \sim r$, which is in agreement with the Regge phenomenology and string picture of hadrons.

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