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Determination of Wolfenstein Parameters in NN Scattering Directly from Observables

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Abstract The Wolfenstein parameters are for the first time obtained analytically in terms of observables. It is shown that a set of ten nucleon–nucleon (NN) observables, which contains polarization observables together with the differential cross section, determines uniquely the solution for Wolfenstein parameters except for a common insignificant phase. Using such analytical solutions one expects to get more accurate theoretical parameters for the potential models by χ^2 fitting to the resulting Wolfenstein parameter data than the standard manner of a phase-shift analysis. An example of fixing a unique set of physical solutions for the Wolfenstein parameters from a set of 16 solutions based on nine observables alone and adding one more observable is illustrated using pseudo data generated by the CD Bonn potential.

1 Introduction

Chiral effective field theory [1] provides an approach to NN potentials which systematically improves by increasing the order in the chiral expansion. Like in standard meson-exchange potentials, which are mostly based on the one-boson exchange picture, the chiral NN potentials contain the whole list of spin-momentum operators, accompanied by analytically given scalar functions. Free parameters therein are obtained by fitting theoretical predictions to experimental observables. The standard manner for that fitting procedure relies on a phase shift analysis [2] of the data.

Two kinds of χ^2 fitting have been applied. In one approach, a direct fitting of theoretical observables, $\mathcal{O}^{th}$, to experimental ones, $\mathcal{O}^{exp}$, is performed and χ^2 minimized:

$$\chi^2 = \sum_i |\mathcal{O}_i^{exp} - \mathcal{O}_i^{th}|^2 w_i. \quad (1)$$

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The weights w_i are given by the inverse of the squared experimental errors. In the second approach, another χ^2 is minimized, namely the difference between δ_i^{exp} from a phase-shift analysis [2] and theoretical ones, δ_i^{th} , predicted by a NN potential:

$$\chi_{\text{phase}}^2 = \sum_i |\delta_i^{exp} - \delta_i^{th}|^2. \quad (2)$$

Usually no weights are attached.

A new kind of fitting, proposed here, is based directly on the Wolfenstein parameters W_k occurring in the on-shell NN M -matrix

$$M = \sum_{k=1}^5 W_k \hat{O}_k. \quad (3)$$

The $\hat{O}_k$ are spin-momentum operators shown below in Sect. 2.

If one would have a unique and analytical solution for the Wolfenstein parameters $W_k^{exp} = W_k^{exp}(\mathcal{O}_1, \mathcal{O}_2, \mathcal{O}_3, \dots)$ in terms of some set of observables the fitting could be performed directly on the level of the Wolfenstein parameters:

$$\chi_{new}^2 = \sum_i |W_i^{exp} - W_i^{th}|^2 \tilde{w}_i. \quad (4)$$

Such χ^2 fitting should be more precise than the approach of Eq. (1).

In Sect. 2, we will present an exact analytical solution for the Wolfenstein parameters in terms of a set of observables. This will be illustrated based on pseudo data generated by the CD Bonn potential in Sect. 3.

2 Solution for Wolfenstein Parameters

The Wolfenstein parameters W_k for NN scattering are labeled as a, c, g, h and m and they parametrize linearly the NN M matrix [3,4]:

$$M = a + c(\sigma^{(1)} + \sigma^{(2)})\hat{N} + m(\sigma^{(1)}\hat{N})(\sigma^{(2)}\hat{N}) + (g + h)(\sigma^{(1)}\hat{P})(\sigma^{(2)}\hat{P}) + (g - h)(\sigma^{(1)}\hat{K})(\sigma^{(2)}\hat{K}), \quad (5)$$

where $\sigma^{(1)}$ and $\sigma^{(2)}$ are the standard Pauli matrices for nucleon 1 and 2, respectively, and three unit momenta $\hat{K}$, $\hat{P}$ and $\hat{N}$ are defined through the initial, $\mathbf{q}$, and final, $\mathbf{q}'$, relative momenta of nucleons 1 and 2 as

$$\hat{K} \equiv (\mathbf{q}' - \mathbf{q})/|\mathbf{q}' - \mathbf{q}|, \quad \hat{P} \equiv (\mathbf{q}' + \mathbf{q})/|\mathbf{q}' + \mathbf{q}|, \quad \hat{N} \equiv (\mathbf{q} \times \mathbf{q}')/|\mathbf{q} \times \mathbf{q}'|. \quad (6)$$

Knowledge of the M -matrix provides theoretical observables $\mathcal{O}^{th}$ in Eq. (1). The spin averaged differential cross section I_0 and the nucleon polarization P_0 in the final state when the initial state is unpolarized, are given as

$$I_0 = \frac{1}{4} \text{Tr}[MM^\dagger], \quad I_0 P_0 \hat{N} = \frac{1}{4} \text{Tr}[MM^\dagger \sigma]. \quad (7)$$

The depolarizations D and R are expressed as

$$I_0 D = \frac{1}{4} \text{Tr}[M(\sigma \hat{N})M^\dagger(\sigma \hat{N})], \quad I_0 R = \frac{1}{4} \text{Tr}[M(\sigma \hat{N} \times \hat{q})M^\dagger(\sigma \hat{K})], \quad (8)$$

and for further observables see [5]. Any NN observable is expressed by the M matrix and nucleon spin operators $\sigma^{(1)}$ and $\sigma^{(2)}$ [3–5]. Due to (5) all observables can also be expressed in terms of the Wolfenstein parameters W_k . For instance [5]

Fig. 1 The way in which the physical solution is obtained from all 16 possible solutions. We choose two typical energies described by the “+” (“x”) line for $E_{lab} = 10$ MeV ($E_{lab} = 100$ MeV). On the y-axis the number of the set of unique physical analytical solution is given out of the 16 sets of allowed solutions. It is the solution which reproduces an additional observable C_{PP} or C_{PK} . For details see text

$$\begin{aligned}
 I_0 &= |a|^2 + |m|^2 + 2|c|^2 + 2|g|^2 + 2|h|^2, \\
 I_0 P_0 &= 2\Re\{c^*(a + m)\}, \\
 I_0 D &= |a|^2 + |m|^2 + 2|c|^2 - 2|g|^2 - 2|h|^2, \\
 I_0 R &= \cos \frac{\theta}{2} (|a|^2 - |m|^2 + |g - h|^2 - |g + h|^2) - \sin \frac{\theta}{2} \Im\{2c(a^* - m^*)\}.
 \end{aligned} \tag{9}$$

The inverse problem, going from observables to the Wolfenstein parameters, requires the solution of a complex, nonlinear system of coupled equations. Without loosing generality, the phase of one Wolfenstein parameter can be put to zero. We take c as such parameter. The full solution together with details will be presented in [6]. Here we show only the solution for the parameter c which is given as

$$\begin{aligned}
 c_{\pm}^2 &= (\alpha\delta^4 + \beta\delta^4 + \gamma\delta^4 + \alpha^3\delta^2 + \beta^3\delta^2 - \alpha\beta^2\delta^2 - \alpha\epsilon^2\delta^2 - \beta\epsilon^2\delta^2 + \gamma\epsilon^2\delta^2 \\
 &\quad - \alpha^2\beta\delta^2 + \alpha^2\gamma\delta^2 + \beta^2\gamma\delta^2 - 2\alpha\beta\gamma\delta^2 + \alpha^3\epsilon^2 + \beta^3\epsilon^2 - \alpha\beta^2\epsilon^2 - \alpha^2\beta\epsilon^2 \\
 &\quad - \alpha^2\gamma\epsilon^2 - \beta^2\gamma\epsilon^2 + 2\alpha\beta\gamma\epsilon^2 \pm 2\Gamma) / \{4((\alpha - \beta)^2 + \delta^2)^2\}
 \end{aligned} \tag{10}$$

where

$$\begin{aligned}
 \Gamma^2 &\equiv (\alpha - \beta)^2 \delta^2 \epsilon^2 \left(-\delta^4 - \gamma^2 \delta^2 - \gamma^2 \epsilon^2 + 2\beta\gamma\epsilon^2 + 4\alpha^3\beta - \beta^2(\gamma^2 + \delta^2 + \epsilon^2) \right. \\
 &\quad \left. - \alpha^2(8\beta^2 + \gamma^2 + \delta^2 + \epsilon^2) + 2\alpha(2\beta^3 + (\gamma^2 + 3\delta^2 - \epsilon^2)\beta + \gamma\epsilon^2) \right),
 \end{aligned} \tag{11}$$

and α , β , γ , δ and ϵ are defined as

$$\begin{aligned}
 \alpha &\equiv \frac{1}{4} (I_0 + I_0 D) + \frac{I_0}{4} \left(\cos \frac{\theta}{2} (2R + (A' - R) \cos \theta) + \sin \frac{\theta}{2} (2R' + (A + R') \cos \theta) \right), \\
 \beta &\equiv \frac{1}{4} (I_0 + I_0 D) - \frac{I_0}{4} \left(\cos \frac{\theta}{2} (2R + (A' - R) \cos \theta) + \sin \frac{\theta}{2} (2R' + (A + R') \cos \theta) \right), \\
 \gamma &\equiv 2I_0 C_{NN} - I_0 A_{NN}, \\
 \delta &\equiv \sin \frac{\theta}{2} I_0 R + \cos \frac{\theta}{2} I_0 A, \\
 \epsilon &\equiv I_0 P_0.
 \end{aligned} \tag{12}$$

This is the first time that analytical expressions have been obtained for the Wolfenstein parameters in terms of nine observables (I_0 , P_0 , D , R , R' , A , A' , C_{NN} and A_{NN}). The nonlinear system of coupled equations provides 16 sets of solutions, among which only one must be a unique physical solution. That unique set of Wolfenstein parameters can be selected by using one additional observable (for instance, C_{PP} or C_{PK}) and requiring that all resulting observables must be continuous in the scattering angle.

3 Example Calculation and Outlook

We would like to show how well the unique set of physical solutions can be selected. We use the CD Bonn potential (np force only) to generate observables. As mentioned in Sect. 2, at each angle we have 16 sets of solutions derived from 9 specific observables. These sets can be numbered from 1 to 16. An additional observable, different from the above nine observables, can be used to pick-up at each angle the unique set of physical solutions. In Fig. 1, we demonstrate that at two energies $E_{lab} = 10$ and 100 MeV. That additional observable (for instance, C_{PP} or C_{PK}) enforces, through its continuity in the angle, one unique solution. In the case of 10 MeV, the 12th solution reproduces that additional observable from 0° to 120° . From 120° to 180° it is the 9th set of solutions which must be taken. In the case of 100 MeV one sees in Fig. 1 that a complex dependence for the number of the physical set on the scattering angle results. Despite such rapid jumps between different solutions which appear for different angles, all observables are smooth functions of the angle.

The three-body force at NNNLO in the chiral expansion [7] has many contributing terms which, however, depend on the same low-energy-constants (LEC's) as the chiral NNLO NN potential. We expect that the new approach by Eq. (4) will provide more precise values for those LEC's.

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