Erratum to: Multiple sign-changing solutions for a semilinear Neumann problem and the topology of the configuration space of the domain boundary

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We would like to modify the following three parts.

First, since it seems difficult to show $\varepsilon_0/2 \leq \|\varphi(t_2, u) - \varphi(t_1, u)\|$ in the proof of Lemma 32, we modify as follows:

(E.1) Change the definition of $\mathcal{M}_d$ in p. 342 to $\mathcal{M}_d = \{u \in H_d : \Phi_d(u) \leq 3c_0, u^+, u^- \in \mathcal{N}_d\}$.

(E.2) Remove the assumption

$$|(\nabla \Phi_{d_n}(u_n), u_n)_{H_{d_n}}| \leq 1 \quad \text{for all } n \in \mathbb{N} \quad (0.1)$$

from Proposition 5.

(E.3) Change the definition of $\mathcal{U}_{d,\varepsilon}$ in p. 344 to $\mathcal{U}_{d,\varepsilon} = \{u \in H_d : \text{dist}_{H_d}(u, \mathcal{M}_d) \leq \varepsilon\}$,

(E.4) Remove Lemma 22.

Consequently, we have the following.

- (i) Even without the assumption (0.1) in the proof of Proposition 5, we can show the boundedness of $\{\|u_n\|_{H_{d_n}}\}$. Indeed, since we can choose $\{\tilde{u}_n\}$ such that $\tilde{u}_n \in \mathcal{M}_{d_n}$ and $\|u_n - \tilde{u}_n\|_{H_{d_n}} \rightarrow 0$, we have $\Phi_{d_n}(\tilde{u}_n) \leq 3c_0$ for each $n \in \mathbb{N}$, and hence $\{\|\tilde{u}_n\|_{H_{d_n}}\}$ is bounded by Lemma 17. From $\|u_n - \tilde{u}_n\|_{H_{d_n}} \rightarrow 0$, $\{\|u_n\|_{H_{d_n}}\}$ is also bounded.
- (ii) As a result of (i), the proofs in Lemmas 23 and 24 work without the assumption (0.1).

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- (iii) In the proof of Lemma 26, we do not need the assumption (0.1) or Lemma 22 either. Indeed, since we have changed the definition of $\mathcal{M}_d$, we can show the boundedness of $\{\|u_n\|_{H_{d_n}}\}$ by a similar way as in (i).
- (iv) In order to show $\mathcal{A}$ is bounded, Lemma 22 was used in the last line of p. 349. However, we can show its boundedness by Lemma 15.
- (v) Then the proof of Lemma 32 works, i.e., we can show $\varepsilon_0/2 \leq \|\varphi(t_2, u) - \varphi(t_1, u)\|$.

Next, it seems difficult to show the existence of $\{v_n\} \subset \text{Inv}(\Phi_d^{\mu_0} \setminus \mathcal{E}_{T_0}) \setminus U_1$ satisfying $\Phi_d(v_n) \rightarrow c$ and $(1 + \|v_n\|_{H_d})\|\nabla \Phi_d(v_n)\|_{H_d} \rightarrow 0$ in the proof of Proposition 6. Indeed, from $\frac{2}{n} \geq \frac{1}{16} \int_0^{t_n} \theta(\varphi(t, u_n)) \min \left\{ 1, (1 + \|\varphi(t, u_n)\|_{H_d})^2 \|\nabla \Phi_d(\varphi(t, u_n))\|_{H_d}^2 \right\} dt$ in p. 351, we might only extract $\{s_n\}$ such that $s_n \in [0, t_n]$ and $\theta(\varphi(s_n, u_n)) \rightarrow 0$, which only might show the existence of $\{w_n\} \subset H_d \setminus \tilde{H}_d$ with $\|\varphi(s_n, u_n) - w_n\|_{H_d} \rightarrow 0$. However, it might hold that $\Phi_d(w_n) \rightarrow \infty$. In this case, it seems difficult to obtain a contradiction. So we modify as follows:

- (E.5) We change the definition of θ_1 in p. 348 to $\theta_1(u) = \min\{\text{dist}_{H_d}(u, \Phi_d^{3c_d} \setminus \tilde{H}_d), 1\}$ and we define θ and W as in the paper.
- (E.6) Instead of Lemma 29, we consider that φ is a semiflow such that $\varphi : [0, \infty) \times \{u \in H_d : \Phi_d(u) < 3c_d\} \rightarrow \{u \in H_d : \Phi_d(u) < 3c_d\}$ defined by $\varphi(0, u) = u$, $\partial\varphi/\partial t(t, u) = -W(\varphi(t, u))$ for $(t, u) \in [0, \infty) \times H_d$ with $\Phi_d(u) < 3c_d$. The proof of Lemma 29 works for the existence such φ .
- (E.7) We change the definition of $\mathcal{E}_T$ in p. 349 to $\mathcal{E}_T = \{u \in H_d : \Phi_d(u) < 3c_d \text{ and there exists } t \in [0, T] \text{ such that } \Phi_d(t, u) \in \mathcal{D}_{d, a_0} \cup \Phi_d^{c_d - \delta_0}\}$.
- (E.8) We change Lemma 31 as follows: For each $T > 0$, $\Phi_d^{\mu_0} \cup \mathcal{E}_T$ is a strictly positively invariant, closed subset of $\{u \in H_d : \Phi_d(u) < 3c_d\}$ and $\mathcal{E}_T$ is a strictly positively invariant, closed subset of $\Phi_d^{\mu_0} \cup \mathcal{E}_T$.

Finally, in the statement of Theorem, it was said that if Ω is bounded then there is at least one other sign-changing solution which has at most four nodal domains. The proof in the last section works for the existence of at least one other sign-changing solution, but it seems difficult to show that its nodal domains are at most four. Because it seems difficult to show $l([0, 2] \times C_\rho(\partial\Omega) \times I_b^2) \subset \Phi_d^{\mu_1}$, which was written in p. 355, 1.17 $\uparrow$. In the case $2 \max\{c_{0,+}, c_{0,-}\} < 3 \min\{c_{0,+}, c_{0,-}\}$, letting x_* be a point in Ω such that $\text{dist}_{\mathbb{R}^N}(x_*, \partial\Omega) \geq 5\rho\bar{\rho}$ and $|x_* - x_\sharp| \geq 5\rho\bar{\rho}$ and by the same proof, we can show that the solution has at most four nodal domains. In the case $2 \max\{c_{0,+}, c_{0,-}\} \geq 3 \min\{c_{0,+}, c_{0,-}\}$, letting $k \in \mathbb{N}$ be the least integer such that $2 \max\{c_{0,+}, c_{0,-}\} < k \min\{c_{0,+}, c_{0,-}\}$, we can only show that the other solution has at most $k + 1$ nodal domains.