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*Erratum***Two-bubble solutions
in the super-critical Bahri-Coron's problem**

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Professor O. Rey showed us a gap in our paper [1]. More precisely, he pointed us out that estimate (5.5), and consequently (5.11), are valid for dimensions $3 \leq N \leq 6$, if we use the weighted L^∞ norm $\|\cdot\|_*$ introduced in page 122.

We can recover all the results of our paper [1] and for any dimension $N \geq 3$, if we modify the definition of the $*$ -norm. The consequent changes inside the body of the paper are relatively minor, but necessary. We will devote this erratum to give the new definition of the $*$ -norm and the complete list of the main changes inside the paper.

The definition of $*$ -norm, introduced at page 122 in [1] should be

$$\|\psi\|_* = \sup_{x \in \Omega_\varepsilon} |\omega^{-\beta}(x)\psi(x)| + |\omega^{-(\beta + \frac{1}{N-2})}(x)\nabla\psi(x)|,$$

where

$$\omega(x) = \left((1 + |x - \xi'_1|^2)^{-\frac{N-2}{2}} + (1 + |x - \xi'_2|^2)^{-\frac{N-2}{2}} \right)$$

and $\beta = 1$ if $N = 3$ and $\beta = \frac{2}{N-2}$ if $N \geq 4$.

The statement of Lemma 4.1 still holds true. We just have to remark that, since ϕ_ε can be written as in formula (4.10), then $\phi_\varepsilon \in C^1$ and

$$\begin{aligned} & \partial_{x_j} \phi_\varepsilon(x) - (p + \varepsilon) \int_{\Omega_\varepsilon} \partial_{x_j} G_\varepsilon(x, y) V^{p+\varepsilon-1} \phi_\varepsilon dy \\ &= - \int_{\Omega_\varepsilon} \partial_{x_j} G_\varepsilon(x, y) h_\varepsilon dy \\ & \quad - \sum c_{ij} \int_{\Omega_\varepsilon} V_i^{p-1} Z_{ij} \partial_{x_j} G_\varepsilon(x, y) dy \quad x \in \Omega_\varepsilon, \end{aligned}$$

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where G_ε denotes the Green's function of Ω_ε . Hence, we get the following estimates

$$\begin{aligned}
 & \int_{\Omega_\varepsilon} \partial_{x_j} G_\varepsilon(x, y) |h_\varepsilon| dy \\
 & \leq \|h_\varepsilon\|_{**} C \int_{\mathbb{R}^N} \frac{1}{|x-y|^{N-1}} \left((1+|y-\xi'_1|^2)^{-2} + (1+|y-\xi'_2|^2)^{-2} \right) dy \\
 & \leq C \|h_\varepsilon\|_{**} |\log \varepsilon|^m \omega^{\beta+\frac{1}{N-2}}(x); \\
 & \left| \sum c_{ij} \int_{\Omega_\varepsilon} V_i^{p-1} Z_{ij} \partial_{x_j} G_\varepsilon(x, y) dy \right| \\
 & \leq C (\|\phi_\varepsilon\|_\rho + \|h_\varepsilon\|_{**}) \sum \int_{\mathbb{R}^N} \frac{1}{|x-y|^{N-1}} \left((1+|y-\xi'_i|^2)^{-\frac{N+3}{2}} \right) \\
 & \leq C (\|\phi_\varepsilon\|_\rho + \|h_\varepsilon\|_{**}) \omega^{\beta+\frac{1}{N-2}}(x);
 \end{aligned}$$

$$\int_{\Omega_\varepsilon} \partial_{x_j} G_\varepsilon(x, y) V^{p+\varepsilon-1} |\phi_\varepsilon| dy \leq C \|\phi_\varepsilon\|_\rho \omega^{\beta+\frac{1}{N-2}}(x).$$

So that we have

$$|\partial_{x_j} \phi_\varepsilon(x)| \leq C (\|\phi_\varepsilon\|_\rho + \|h_\varepsilon\|_{**}) |\log \varepsilon|^m \omega^{\beta+\frac{1}{N-2}}(x).$$

Formula (4.11) should be

$$|\phi_\varepsilon(x)| \leq C (\|\phi_\varepsilon\|_\rho + \|h_\varepsilon\|_{**}) |\log \varepsilon|^m \omega^\beta(x).$$

Finally, the last formula from above at page 125 should read as

$$\|\phi_\varepsilon\|_* \leq C (\|\phi_\varepsilon\|_\rho + \|h_\varepsilon\|_{**}) |\log \varepsilon|^m,$$

from which the conclusion of the lemma follows.

The estimates given at the middle of page 129 should be the following.

Assume that $N > 6$. In the region where $\text{dist}(y, \partial\Omega_\varepsilon) \geq \delta \varepsilon^{-\frac{1}{N-2}}$, for some $\delta > 0$, then $V(y) \geq \alpha_\delta \bar{V}(y)$ for some $\alpha_\delta > 0$; hence in this region, we have

$$|\bar{V}^{-\frac{4}{N-2}} N_\varepsilon(\eta)| \leq C \bar{V}^{2\beta-1} \|\eta\|_*^2 \leq C \varepsilon^{2\beta-1} \|\eta\|_*^2.$$

On the other hand, when $\text{dist}(y, \partial\Omega_\varepsilon) \leq \delta \varepsilon^{-\frac{1}{N-2}}$, we have

$$V(y) = C(y) \varepsilon^{\frac{N-1}{N-2}} \text{dist}(y, \partial\Omega_\varepsilon) + O(\delta^2 \varepsilon + \varepsilon^{\frac{N+1}{N-2}})$$

with $C(y) \geq C_0 > 0$; hence,

$$V(y) \geq \frac{C_0}{2} \varepsilon^{\frac{N-1}{N-2}} \text{dist}(y, \partial\Omega_\varepsilon)$$

as $\text{dist}(y, \partial\Omega_\varepsilon) \leq \delta \varepsilon^{-\frac{1}{N-2}}$, provided that $\delta > 0$ is chosen small enough. These estimates are a consequence of the fact that the Green function of the domain Ω vanishes linearly with respect to $\text{dist}(x, \partial\Omega)$ as $x \rightarrow \partial\Omega$.

Hence we can conclude that, if $\text{dist}(y, \partial\Omega_\varepsilon) \leq \delta\varepsilon^{-\frac{1}{N-2}}$, then

$$\begin{aligned} |\bar{V}^{-\frac{4}{N-2}} N_\varepsilon(\eta)| &\leq \bar{V}^{-\frac{4}{N-2}} V^{p-2} \|\eta\|^2 \\ &\leq C \bar{V}^{-\frac{4}{N-2}} \left(\varepsilon^{\frac{N-1}{N-2}} \text{dist}(y, \partial\Omega_\varepsilon) \right)^{p-2} \text{dist}(y, \partial\Omega_\varepsilon)^2 \|\nabla \eta(y)\|^2 \\ &\leq C \varepsilon^{-\frac{4}{N-2} + \frac{N-1}{N-2}(p-2) - \frac{p}{N-2} + 2\beta + \frac{2}{N-2}} \|\eta\|_*^2 \leq C \varepsilon^{2\beta-1} \|\eta\|_*^2. \end{aligned}$$

Formula (5.5), and hence (5.11), still hold true.

The first formula from above in page 131 should be

$$\begin{aligned} &\|N_\varepsilon(\psi + \phi_1) - N_\varepsilon(\psi + \phi_2)\|_{**} \leq \\ &\begin{cases} \bar{V}^{2\beta-1} (\|\phi_1\|_* + \|\phi_2\|_* + \|\psi\|_*) \|\phi_1 - \phi_2\|_* & \text{if } N \leq 6 \\ \varepsilon^{2\beta-1} (\|\phi_1\|_* + \|\phi_2\|_* + \|\psi\|_*) \|\phi_1 - \phi_2\|_* & \text{if } N > 6 \end{cases} \\ &\leq \varepsilon^{\min\{2\beta, 1\}} \|\phi_1 - \phi_2\|_*. \end{aligned}$$

The second formula from below at page 131 should be

$$\begin{aligned} &\bar{V}^{-\frac{4}{N-2} + \beta} |D_{\bar{\phi}} N_\varepsilon(\phi + \psi)| \\ &\leq C \begin{cases} \bar{V}^{2\beta-1} \|\phi + \psi\|_* & \text{if } N \leq 6 \\ \varepsilon^{2\beta-1} \|\phi + \psi\|_* & \text{if } N > 6 \end{cases} \leq C \varepsilon^{\min\{2\beta, 1\}}. \end{aligned}$$

The forth formula from below at page 132 should be

$$\begin{aligned} |(D_{\xi'} N)(\xi', A, \bar{\phi})| &\leq C \bar{V}^{\frac{N-1}{N-2}} |(V + \bar{\phi})_+^{p+\varepsilon-1} - V^{p+\varepsilon-1} - (p + \varepsilon - 1) V^{p+\varepsilon-2} \bar{\phi}| \\ &\leq C \begin{cases} \bar{V}^{\frac{5}{N-2} + \varepsilon + \beta} \|\bar{\phi}\|_* & \text{if } N \leq 6 \\ \varepsilon^{\frac{5}{N-2} + \varepsilon + \beta} \|\bar{\phi}\|_* & \text{if } N > 6. \end{cases} \end{aligned}$$

Reference

1. del Pino, M., Felmer, P., Musso, M.: Two-bubble solutions in the super-critical Bahri-Coron's problem. *Calc. Var.* **16**, 113–145 (2003)