Editorial

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This special issue contains papers of several authors dealing with the concept of fuzzy transform (F -transform) established by I. Perfilieva in her famous paper published in 2006 (cfr., Fuzzy Sets and Systems 157, pp. 993–1023), both from the mathematical and from the application point of view. Recall that the concept of F -transform is very well mathematically founded and has deep non-trivial theory. Moreover, it has a lot of profound applications in image and signal processing (up- (down-) scaling, reconstruction, edge detection, fusion, registration, etc.), in time series analysis (trend extraction and reduction of noise), in the big data processing (pattern recognition), in the numerical analysis and elsewhere. The F -transform is thus one of the most distinguished examples, why fuzzy methods are useful and effective on all levels of data processing.

Below is a brief description of each paper:

- in the paper titled “An Adaptive Image Filter Based on the Fuzzy Transform for Impulse Noise Reduction” by T. Schuster and P. Sussner, an impulsive noise in a digital image is preliminary dealt via a median filter. After an application of the direct fuzzy transform to the resulting image, the pixel values corresponding to locations flagged by the fuzzy rule-based noise detector are restored via the inverse fuzzy transform. Comparison with other image filters for impulsive noise reduction is given.
- in the paper titled “A new representation for inverse fuzzy transform and its application” by A. Khastan, a new formula for basic functions of fuzzy transform is presented. It provides new approximation properties of the inverse fuzzy transform. In particular, properties of sinusoidal basic functions give a new fuzzy-based method for solving nonlinear Fredholm integral equations.
- in the paper titled “Spaces with fuzzy partitions and fuzzy transform” by J. Močkoř, a category of spaces with fuzzy partitions is given as a ground category for the F -transform constructions. Axioms of fuzzy transform systems are defined and relationships between the categories of Kuratowski closure and interior operators and the category of fuzzy preorder relations are investigated.
- in the paper titled “Fuzzy Transforms Prediction in Spatial Analysis and its Application to Demographic Balance Data” by F. Di Martino and S. Sessa, a new prediction algorithm based on fuzzy transforms for forecasting problems in spatial analysis is suggested. As an example, the authors test their method by exploring the demographic balance data measured every month in the period 01/01/2003–31/10/2014 in all the municipalities of “Cilento and Vallo di Diano” National Park located in the district of Salerno (Italy). They predict the value of the parameters “birthrate” and “deathrate” in November 2014. Moreover, a fuzzification process is given to establish the thematic map of the errors calculated between the real data and the predicted data. The thematic maps are constructed in a GIS environment.
- in the paper titled “Trigonometric Fm-transform and its approximative properties” by R. Alikhani et al., the concept of F -transform is generalized, via trigonometric

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- polynomials up to m degree, $m \geq 0$, with the concept of F^m -transform, whose basic functions are sinusoidal. Applying the Gram–Schmidt procedure in order to achieve the orthogonal system, an explicit representation for them for arbitrary m is obtained. The approximation and convergence properties of the direct and inverse F^m -transforms are discussed and their applicability are illustrated by examples.
- in the paper titled “Weighted Transform and Approximation of Some Functions on Unbounded Sets” by S. Jahedi, F. Javadi and M. J. Mehdipour, a fuzzy transform via the concept of a weighted partition is given. Thus real-valued continuous functions defined on unbounded interval $[0, \infty)$ and vanishing at infinity can be approximated with arbitrary precision. The authors characterize approximation of integrable functions on $[0, \infty)$.
 - in the paper titled “On the relationship among F -transform, fuzzy rough set and fuzzy topology” by I. Perfilieva et al., the objective is to associate the concepts of fuzzy rough sets and fuzzy topologies/co-topologies with the F -transforms. Mainly the notions of the direct $F^\uparrow$ - and $F^\downarrow$ -transforms are extended and shown to be particular cases of the upper and lower fuzzy approximation operators.
 - in the paper titled “Continuous and Discrete Higher Degree F -transforms Based on B-splines” by M. Kokainis and S. Asmuss, continuous and discrete higher-degree fuzzy transforms (F -transforms with polynomial components) with respect to a generalized fuzzy partition are given by B-splines that allow to improve the quality of approximation of smooth functions.
 - in the paper titled “Pattern matching: overview, benchmark and comparison with F -transform general matching algorithm” by P. Hurtik and P. Števuliaková, a general soft computing algorithm for pattern matching based on the fuzzy transform is proposed together with its version working on graphic cards.
 - in the paper titled “Approximation by Multivariate Higher Degree F -transform Based on B-splines” by M. Kokainis and S. Asmuss, integral and discrete versions of the direct and inverse higher-degree fuzzy transforms of multivariate functions are proposed. The goal is generalization of the univariate F -transform to the multidimensional case with respect to a generalized fuzzy partition given by B-splines. The use of multivariate B-splines as the generating functions of multidimensional fuzzy partition allows to improve the quality of approximation of multivariate functions and their derivatives.
 - in the paper titled “ F -transform Based Shooting Method for Nonlinear Boundary Value Problems” by I. Perfilieva, P. Števuliaková and R. Valášek, a class of the ordinary two-point boundary value problems for nonlinear second-order ordinary differential equations is studied. The second-degree F -transform for solving of the corresponding initial value problem is applied. Experiments demonstrate effectiveness of the proposed F -transform-based method, its stability and ability to “ignore noise.”
 - in the paper titled “Multivariate Fuzzy Transform of Complex-Valued Functions Determined by Monomial Basis” by L. Nguyen, M. Holčápek and V. Novák, the multivariate fuzzy transform of higher degree of complex-valued functions is introduced. Apart from the orthogonal bases of the multivariate complex polynomials of weighted Hilbert spaces that are derived by the Gram–Schmidt orthogonalization process, the authors propose to compute the multivariate fuzzy transform components using a simple matrix calculus with the help of the monomial bases. By this novel approach, two types of the upper bound of the approximation error both of the multivariate complex-valued functions and of their partial derivatives (the latter by the multivariate higher-degree fuzzy transform) are derived. The results are demonstrated on examples.
 - in the paper titled “An extension of F -transforms to more general data. Potential applications” by N. Madrid, the author addresses handicap of fuzzy transforms when they are to be applied to data in a broader sense: They are by definition applicable only to functions. This paper presents an extension of the definition of the F -transforms to a more general structure of the data that need not be functions. Potential applications of this novel extension are also presented.

We hope that this special issue will attract the attention of the readers of this journal. The included papers illustrate the wealth of the world in which fuzzy transform lives per se: both pure mathematical theory and enormous potential for applications. The fuzzy transform thus has a distinguished position among the soft computing methods. We also thank the Editorial Management of the Soft Computing journal for giving us the opportunity to edit this issue.

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