ADDENDUM
COMMENTARY FOR
“ON PLANARITY OF COMPACT, LOCALLY CONNECTED,
METRIC SPACES”

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We add some historical remarks concerning our paper, “On planarity of compact, locally connected, metric spaces,” in this journal **31** (2011), no. 3, 365–376.

Our paper [5] in this journal extends Kuratowski’s classical planarity condition to compact, locally connected, metric spaces. It has been brought to our attention that its main considerations have previously appeared. The principal source is Claytor [3], who proves a version equivalent to ours. In place of our “generalized thumbtacks” he provides two specific examples of graph-like continua. The equivalence is that there is a generalized thumbtack in a compact, locally connected, metric space M if and only if one of the two examples occurs in M .

In their work [1], Chávez, Fernández-Bayort, Quintero, and Villar point out that Moise [4] gives a short proof of Claytor’s original theorem [2], reproved by Thomassen [6], for the planarity of a 2-connected, compact, locally connected, metric space. As Moise points out, his proof involves the use of a substantial result of Bing. Furthermore, Chávez et al show that these theorems generalize to locally compact, locally connected, metric spaces.

In addition, Chávez et al discuss the “local 2-connection” condition from Thomassen’s paper [7] and prove theorems related to special cases of the main theorem of [7], although the results of [7] are understood by us to be novel, requiring only that the space under consideration not contain an infinite complete graph.

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In retrospect, it is not surprising that such fundamental theorems have been proved before. We hope and feel that our graph-theoretic approach brings additional perspective to the results. We appreciate the cooperation of Tomás Fernández-Bayort in directing us to relevant references.

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