ERRATUM

PUNCTURED COMBINATORIAL NULLSTELLENSÄTZE

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In this erratum we correct a claim made in [1, Corollary 4.2].

Let $\mathbb{F}$ be a field and let f be a polynomial in $\mathbb{F}[X_1, X_2, \dots, X_n]$. For $i = 1, \dots, n$, let D_i and S_i be finite non-empty subsets of $\mathbb{F}$, where $D_i \subset S_i$, and define

$$g_i(X_i) = \prod_{s \in S_i} (X_i - s) \text{ and } l_i(X_i) = \prod_{d \in D_i} (X_i - d).$$

In [1] the authors prove the following theorem.

Theorem 4.1. *If f has a zero of multiplicity at least t at all the common zeros of $g_1, g_2, \dots, g_n$, except at at least one point of $D_1 \times D_2 \times \dots \times D_n$ where it has a zero of multiplicity less than t , then there are polynomials h_τ in $\mathbb{F}[X_1, X_2, \dots, X_n]$, satisfying $\deg(h_i) \leq \deg(f) - \sum_{i \in \tau} \deg(g_i)$, and a non-zero polynomial u satisfying $\deg(u) \leq \deg(f) - \sum_{i=1}^n (\deg(g_i) - \deg(l_i))$, such that*

$$f = \sum_{\tau \in T(n,t)} g_{\tau(1)} \dots g_{\tau(t)} h_\tau + u \prod_{i=1}^n \frac{g_i}{l_i}.$$

Moreover, if there is a point of $D_1 \times D_2 \times \dots \times D_n$ where f is non-zero, then,

$$\deg(f) \geq (t-1) \max_j (|S_j| - |D_j|) + \sum_{i=1}^n (|S_i| - |D_i|).$$

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We then claimed the following corollary.

Corollary 4.2. *If $D_1 \times \cdots \times D_n$ is a grid containing all the points of the grid $S_1 \times \cdots \times S_n$ where f does not vanish and $D_i \subset S_i$ for all i , then f has a term $X_1^{r_1} \cdots X_n^{r_n}$, where $|S_i| - 1 \geq r_i \geq |S_i| - |D_i|$.*

Proof. By Theorem 4.1 we can write

$$f = \sum_{i=1}^n h_i g_i + w,$$

and

$$w = u \prod_{i=1}^n \frac{g_i}{l_i},$$

for some non-zero polynomial u , where the degree in X_i of u is less than $|D_i|$. Therefore the degree in X_i of w is less than $|S_i|$ and at least $|S_i| - |D_i|$. ■

However knowing that w has such a term does not necessarily imply that f has such a term.

For example

$$f = X_1 X_2 (X_1 - X_2) = X_1 (X_1 - 1) X_2 - X_2 (X_2 - a) X_1 + (1 - a) X_1 X_2,$$

is zero on all of $\{0, 1\} \times \{0, a\}$ except $\{1\} \times \{a\}$, where $a \neq 1$.

One can only claim the following corollary.

Corollary. *If $D_1 \times \cdots \times D_n$ is a grid containing all the points of the grid $S_1 \times \cdots \times S_n$ where f does not vanish and $D_i \subset S_i$ for all i , then f has a term $X_1^{r_1} \cdots X_n^{r_n}$, where $r_i \geq |S_i| - |D_i|$.*

References

- [1] S. BALL and O. SERRA: Punctured combinatorial Nullstellensätze, *Combinatorica* **29(5)** (2009), 511–522.

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