Reduction of the number of material parameters by ANN approximation

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Abstract Modern industrial standards require advanced constitutive modeling to obtain satisfactory numerical results. This approach however, is causing significant increase in number of material parameters which can not be easily obtained from standard and commonly known experimental techniques. Therefore, it is desirable to introduce procedure decreasing the number of the material parameters. This reduction however, should not lead to misunderstanding the fundamental physical phenomena. This paper proposes the reduction of the number of material parameters by using ANN approximation. Recently proposed viscoplasticity formulation for anisotropic solids (metals) developed by authors is used as an illustrative example. In this model one needs to identify 28 material parameters to handle particular metal behaviour under adiabatic conditions as reported by Glema et al (J Theor Appl Mech 48:973–1001, 2010), (Int J Damage Mech 18:205–231, 2009) and Sumelka (Poznan University of Technology, Poznan, 2009). As a result of proposed approach, authors decreased the number of material parameters to 19.

Keywords Material parameters · Anisotropy · Viscoplasticity · ANN approximation

1 Introduction

The complexity of mathematical aspect of metal behavior modeling maintain on rising in order to match recent industrial requirements (especially thermo-mechanical coupling including damage). The modeling must reflect micro/nano scale of observation in order to capture physical phenomena crucial for its proper description [6,37]. As a result, the modern constitutive models include high number of material parameters. For example, well established and relatively simple constitutive model proposed by Johnson and Cook [10] requires identification of 14 material parameters to describe metal behavior. More sophisticated models, such as Rusinek–Klepaczko model [30–32] or recently proposed model to analyze hypervelocity projectile-target impact [4] require identification up to 19 (depending on analyzed process) and even 30 material parameters, respectively.

The experimental (or mathematical) identification of such models, if at all possible, can be very expensive (in its common or computational time sense). Hence it is obvious that the reduction of the number of material parameters, without losing particular model generality, is advisable. In order to meet such demands, authors present in this paper the general procedure of using the *artificial neural network* (ANN) for material functions approximation. As an application example of ANN approximation procedure we use recently proposed viscoplasticity formulation for anisotropic solids, showing finally its effects in the reduction of desired material parameters in the model under consideration (from 28 to 19).

This paper is divided into three major parts.

In Sect. 2 the fundamental results of viscoplastic theory formulation for anisotropic solids are presented to understand necessity of further described ANN approximation. In particular we discuss: description of damage phenomena, definition of the body kinematics, constitutive postulates and

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constitutive relations with a complete definition of material functions for adiabatic process.

Section 3 deals with the concept of ANN approximation for selected material functions in the discussed model. The most important result of this section is that we can apply ANN approximation without losing the physical interpretation of all variables in the model comparing to the classical viscoplasticity formulation by Perzyna (cf. [28]).

Finally, in Sect. 4 the material model implementation in a finite element software Abaqus/Explicit by utilizing VUMAT user subroutine is briefly discussed and its application concerning high speed machining (orthogonal cutting of DH-36 steel sample) is presented.

2 Main results of viscoplasticity formulation for anisotropic solids

2.1 General discussion

The material model is formulated in terms of the continuum mechanics, in the framework of the thermodynamical theory of viscoplasticity along with the phenomenological approach [7,29,36]. Formally, the constitutive structure belongs to the class of simple materials with fading memory, and due to its final form and the way of incorporating the fundamental variables, belongs to the rate type materials with internal state variables [38].

The key features of the formulation (for detailed and more general formulation please see [6,35,37]) are: (i) the description is invariant with respect to any diffeomorphism (the obtained model is covariant [13]), (ii) the obtained evolution problem is well-posed, (iii) sensitivity to the rate of deformation, (iv) finite elasto-viscoplastic deformations, (v) plastic non-normality, (vi) dissipation effects (anisotropic description of damage), (vii) thermo-mechanical couplings and (viii) length scale sensitivity.

In the discussed model an important role plays the description of damage. We introduce the second order microdamage tensorial field (as a state variable), denoted by ξ cf. [7,29,35], which reflects the experimentally observed anisotropy of metals in the mathematical (constitutive) model. Such approach enables us to maintain high level of the global damage approximation (GDA) (strain–stress curves fitting from experiment and mathematical model) as well as particularly high level of local damage approximation (LDA) (GDA plus coincidence in: macrodamage initiation time, velocity of macrodamage evolution and the geometry of macrodamage pattern). Let us emphasize that the Euclidean norm of the microdamage field defines the scalar quantity called the *volume fraction porosity* or simply *porosity* [29] while its principal values are proportional to the ratio of the damaged area and the assumed characteristic area of the representative

Fig. 1 The concept of microdamage tensor

volume element [35]. Thus, they indicate damage plane as one perpendicular to maximal principal value of ξ (cf. Fig. 1).

2.2 Adiabatic process

Kinematics

The abstract body is a differentiable manifold. The kinematics of the finite elasto-viscoplastic deformations is governed by the multiplicative decomposition of the total deformation gradient to the elastic and viscoplastic parts [12]

$$\mathbf{F}(\mathbf{X}, t) = \mathbf{F}^e(\mathbf{X}, t) \cdot \mathbf{F}^p(\mathbf{X}, t), \quad (1)$$

where (1) $\mathbf{F} = \frac{\partial \phi(\mathbf{X}, t)}{\partial \mathbf{X}}$ is the deformation gradient, ϕ describes the motion, $\mathbf{X}$ denotes material coordinates, t represents time and $\mathbf{F}^e$, $\mathbf{F}^p$ are elastic and viscoplastic parts of the deformation gradient, respectively.

From the spatial deformation gradient, denoted by $\mathbf{l}$,

$$\mathbf{l}(\mathbf{x}, t) = \frac{\partial \mathbf{v}(\mathbf{x}, t)}{\partial \mathbf{x}}, \quad (2)$$

where $\mathbf{v}$ denotes spatial velocity and $\mathbf{x}$ are spatial coordinates, we obtain

$$\mathbf{l} = \mathbf{d} + \mathbf{w} = \mathbf{d}^e + \mathbf{w}^e + \mathbf{d}^p + \mathbf{w}^p, \quad (3)$$

$$\mathbf{d} = \frac{1}{2}(\mathbf{l} + \mathbf{l}^T), \quad (4)$$

$$\mathbf{w} = \frac{1}{2}(\mathbf{l} - \mathbf{l}^T), \quad (5)$$

where $\mathbf{d}$ is the symmetric part and $\mathbf{w}$ is the antisymmetric part, of $\mathbf{l}$, respectively. Now, assuming that the Euler–Almansi strain is taken as a strain measure and applying Lie derivative we have the fundamental relation

$$\mathbf{d}^b = \mathbf{L}\mathbf{v}(\mathbf{e}^b), \quad (6)$$

and simultaneously

Fig. 2 The strain–stress relations taken from experimental results [15] (red lines) and its ANN approximation (black lines). (Color figure online)

Fig. 3 The strain–stress relations taken from experimental results [15] (red lines) and its ANN approximation/extrapolation (black lines) through the temperature axis. (Color figure online)

$$\mathbf{d}^{e^b} = L_{\mathbf{v}}(\mathbf{e}^{e^b}), \quad \mathbf{d}^{p^b} = L_{\mathbf{v}}(\mathbf{e}^{p^b}), \quad (7)$$

where $L_{\mathbf{v}}$ stands for Lie derivative, $\mathbf{e}$ stands for the Euler–Almansi strain, b indicates that a tensor has all its

indices lowered [13], indices e and p denote elastic and viscoplastic parts, respectively. These relations show that the symmetric part of the spatial deformation gradient $\mathbf{d}$ is directly Lie derivative of the Euler–Almansi strain.

Fig. 4 The strain–stress relations taken from experimental results [15] (red lines) and its ANN approximation/extrapolation (black lines) through the temperature and strains axis. (Color figure online)

Constitutive postulates

Assuming that the balance principles hold, namely: conservation of mass, balance of momentum, balance of moment of momentum and balance of energy and entropy production, we define four constitutive postulates [26,28]:

- (i) Existence of the free energy function ψ . Formally we apply it in the following form

$$\psi = \hat{\psi}(\mathbf{e}, \mathbf{F}, \vartheta; \boldsymbol{\mu}), \quad (8)$$

where $\boldsymbol{\mu}$ denotes a set of internal state variables governing the description of dissipation effects and ϑ represents temperature. It is important to notice, that we have used semicolon to separate the last variable due to its different nature (it introduces a dissipation to the model), without $\boldsymbol{\mu}$ the presented model describes thermoelasticity.

- (ii) Axiom of objectivity (spatial covariance). The material model should be invariant with respect to any superposed motion (diffeomorphism).
- (iii) The axiom of the entropy production. For every regular process the constitutive functions should satisfy the second law of thermodynamics.
- (iv) The evolution equation for the internal state variables vector $\boldsymbol{\mu}$ should be of the form

$$\mathbf{L}_v \boldsymbol{\mu} = \hat{\mathbf{m}}(\mathbf{e}, \mathbf{F}, \vartheta, \boldsymbol{\mu}), \quad (9)$$

where evolution function $\hat{\mathbf{m}}$ has to be determined based on the experimental observations.

Initial boundary value problem

Assuming that the above assumptions hold, the deforming body under adiabatic regime is governed by the following set of equations. They state the initial boundary value problem (IBVP).

Find ϕ , $\mathbf{v}$, ρ , $\boldsymbol{\tau}$, $\boldsymbol{\xi}$, ϑ as functions of t and position $\mathbf{x}$ such that the following equations are satisfied [19,21,22,27]:

- (i) The field equations

$$\begin{aligned} \dot{\phi} &= \mathbf{v}, \\ \dot{\mathbf{v}} &= \frac{1}{\rho_{Ref}} \left(\operatorname{div} \boldsymbol{\tau} + \frac{\boldsymbol{\tau}}{\rho} \cdot \operatorname{grad} \rho - \frac{\boldsymbol{\tau}}{1 - (\boldsymbol{\xi} : \boldsymbol{\xi})^{\frac{1}{2}}} \operatorname{grad} (\boldsymbol{\xi} : \boldsymbol{\xi})^{\frac{1}{2}} \right), \\ \dot{\rho} &= -\rho \operatorname{div} \mathbf{v} + \frac{\rho}{1 - (\boldsymbol{\xi} : \boldsymbol{\xi})^{\frac{1}{2}}} (\mathbf{L}_v \boldsymbol{\xi} : \mathbf{L}_v \boldsymbol{\xi})^{\frac{1}{2}}, \\ \dot{\boldsymbol{\tau}} &= \mathcal{L}^e : \mathbf{d} + 2\boldsymbol{\tau} \cdot \mathbf{d} - \mathcal{L}^{th} \dot{\vartheta} - (\mathcal{L}^e + \mathbf{g}\boldsymbol{\tau} + \boldsymbol{\tau}\mathbf{g}) : \mathbf{d}^p, \\ \dot{\boldsymbol{\xi}} &= 2\boldsymbol{\xi} \cdot \mathbf{d} + \frac{\partial g^*}{\partial \boldsymbol{\tau}} \frac{1}{T_m} \left\langle \Phi^g \left[\frac{I_g}{\tau_{eq}(\boldsymbol{\xi}, \vartheta, \epsilon^p)} - 1 \right] \right\rangle, \\ \dot{\vartheta} &= \frac{\chi^*}{\rho c_p} \boldsymbol{\tau} : \mathbf{d}^p + \frac{\chi^{**}}{\rho c_p} \mathbf{k} : \mathbf{L}_v \boldsymbol{\xi}, \end{aligned} \quad (10)$$

Fig. 5 The flow surfaces through the change of strain rates: experimental results 0.001, 0.1, 3,000 s^{-1} [15] and its ANN extrapolation 0.0001, 500, 10,000 s^{-1}

(ii) The boundary conditions

- (a) Displacement ϕ is prescribed on a part Γ_ϕ of $\Gamma(\mathcal{B})$ and tractions $(\boldsymbol{\tau} \cdot \mathbf{n})^a$ are prescribed on a part Γ_τ of $\Gamma(\mathcal{B})$, where $\Gamma_\phi \cap \Gamma_\tau = \emptyset$ and $\Gamma_\phi \cup \Gamma_\tau = \Gamma(\mathcal{B})$,
- (b) Heat flux $\mathbf{q} \cdot \mathbf{n} = 0$ is prescribed on $\Gamma(\mathcal{B})$,

- (iii) The initial conditions $\phi, \mathbf{v}, \rho, \boldsymbol{\tau}, \boldsymbol{\xi}, \vartheta$ are given for each particle $\mathbf{X} \in \mathcal{B}$ at $t = 0$,

In above equations we have denoted: ρ_{Ref} as a referential density, $\boldsymbol{\tau}$ as the Kirchhoff stress tensor, ρ as a current density, $\mathcal{L}^e$ as an elastic constitutive tensor, $\mathcal{L}^{th}$ as a thermal operator, $\mathbf{g}$ as a metric tensor, $\frac{\partial \mathbf{g}^*}{\partial \boldsymbol{\tau}}$ as the evolution directions for anisotropic microdamage growth processes, T_m as a relaxation time of the mechanical disturbances, I_g as a stress

intensity invariant, τ_{eq} as the threshold stress, χ^* , χ^{**} as the irreversibility coefficients and c_p as a specific heat.

Material functions

For the evolution problem (10) we assume the following:

1. For elastic constitutive tensor $\mathcal{L}^e$

$$\mathcal{L}^e = 2\mu\mathcal{I} + \lambda(\mathbf{g} \otimes \mathbf{g}), \quad (11)$$

where μ, λ are Lamé constants.

2. For thermal operator $\mathcal{L}^{th}$

$$\mathcal{L}^{th} = (2\mu + 3\lambda)\theta\mathbf{g}, \quad (12)$$

Table 1 Material parameters for DH-36 steel - boxed parameters are those being reduced by ANN approximation

$\lambda = 121.154 \text{ GPa}$	$\mu = 80.769 \text{ GPa}$	$T_m = 2.5 \mu\text{s}$	$m_{pl} = 0.13$
$n_1 = 0$	$n_2 = 0.25$	$\chi^* = 1.0$	$\chi^{**} = 0.8$
$\boxed{\kappa_s^*}$	$\boxed{\kappa_s^{**}}$	$\boxed{\kappa_0^*}$	$\boxed{\kappa_0^{**}}$
$\boxed{\delta^*}$	$\boxed{\delta^{**}}$	$\beta^* = 9.0$	$\beta^{**} = 2.0$
$c = 0.067$	$\theta = 10^{-6} \text{ K}^{-1}$	$m_g = 1$	$m_F -$
$b_1 = 0.58$	$b_2 = 0.7$	$b_3 = 0$	$\ L\mathbf{v}\xi_c\ -$
$\xi^{F*} = 0.30$	$\xi^{F**} -$	$c_p = 470 \text{ J/kgK}$	$\rho_{Ref} = 7,800 \text{ kg/m}^3$

Fig. 6 The comparison of strain–stress curves (isothermal and adiabatic) from the experiment and the model for a strain rate $3,000 \text{ s}^{-1}$ and initial temperature of 296 K **Fig. 7** **a** Configuration of the high speed machining; **b** Geometry of the rigid tool

where θ is thermal expansion coefficient.

3. For viscoplastic flow phenomenon $\mathbf{d}^p$ [23,24]

$$\mathbf{d}^p = \Lambda^{vp} \mathbf{p}, \quad (13)$$

where

$$\begin{aligned} \Lambda^{vp} &= \frac{1}{T_m} \left\langle \Phi^{vp} \left(\frac{f}{\kappa} - 1 \right) \right\rangle \\ &= \frac{1}{T_m} \left\langle \left(\frac{f}{\kappa} - 1 \right)^{m_{pl}} \right\rangle, \end{aligned} \quad (14)$$

$$f = \left\{ J_2' + \left[n_1(\vartheta) + n_2(\vartheta) (\xi : \xi)^{\frac{1}{2}} \right] J_1^2 \right\}^{\frac{1}{2}}, \quad (15)$$

$$n_1(\vartheta) = 0, \quad n_2(\vartheta) = n = \text{const.}, \quad (16)$$

$$\kappa = \left\{ \kappa_s(\vartheta) - [\kappa_s(\vartheta) - \kappa_0(\vartheta)] \exp[-\delta(\vartheta) \epsilon^p] \right\}$$

$$\left[1 - \left(\frac{(\xi : \xi)^{\frac{1}{2}}}{\xi_F} \right)^{\beta(\vartheta)} \right], \quad (17)$$

$$\bar{\vartheta} = \frac{\vartheta - \vartheta_0}{\vartheta_0}, \quad \kappa_s(\vartheta) = \kappa_s^* - \kappa_s^{**} \bar{\vartheta},$$

$$\kappa_0(\vartheta) = \kappa_0^* - \kappa_0^{**} \bar{\vartheta}, \quad \delta(\vartheta) = \delta^* - \delta^{**} \bar{\vartheta},$$

$$\beta(\vartheta) = \beta^* - \beta^{**} \bar{\vartheta}, \quad (18)$$

$$\xi^F = \xi^{F*} - \xi^{F**} \left\langle \left(\frac{\|L\mathbf{v}\xi\| - \|L\mathbf{v}\xi_c\|}{\|L\mathbf{v}\xi_c\|} \right)^{m_F} \right\rangle, \quad (19)$$

$$\begin{aligned} \mathbf{p} &= \frac{\partial f}{\partial \boldsymbol{\tau}} \bigg|_{\xi=\text{const}} \left(\left\| \frac{\partial f}{\partial \boldsymbol{\tau}} \right\| \right)^{-1} \\ &= \frac{1}{[2J_2' + 3A^2(\text{tr}\boldsymbol{\tau})^2]^{\frac{1}{2}}} \left[\boldsymbol{\tau}' + A \text{tr}\boldsymbol{\tau} \boldsymbol{\delta} \right], \end{aligned} \quad (20)$$

Fig. 8 The comparison of reaction on tool for cutting depths 50 and 100 μm

and f denotes the potential function [7,25,26,33], κ is the isotropic work hardening–softening function [17, 26], $\boldsymbol{\tau}'$ represents the stress deviator, J_1 , J_2' are the first and the second invariants of Kirchhoff stress tensor and deviatoric part of the Kirchhoff stress tensor, respectively, $A = 2 \left(n_1 + n_2 (\boldsymbol{\xi} : \boldsymbol{\xi})^{\frac{1}{2}} \right)$, ξ^{F*} can be thought as a quasi-static fracture porosity and $\|\mathbf{L}\boldsymbol{\nu}\boldsymbol{\xi}_c\|$ denotes equivalent critical velocity of microdamage. Notice, that Eq. (19) reflects experimental fact, that the fracture porosity changes for fast processes. Such approach is consistent with the, so called, cumulative fracture criterion [2,9,11], which assumes the existence of critical time needed for saturation of microdamage to its fracture limit.

4. For microdamage mechanism we take the additional assumptions [3,7]:

- Velocity of the microdamage growth is coaxial with the principal directions of stress state,
- Only positive (tension) principal stresses induces the growth of the microdamage,

so we have

$$\frac{\partial g^*}{\partial \boldsymbol{\tau}} = \left\langle \frac{\partial \hat{g}}{\partial \boldsymbol{\tau}} \right\rangle \left\| \left\langle \frac{\partial \hat{g}}{\partial \boldsymbol{\tau}} \right\rangle \right\|^{-1}, \quad \text{and} \quad \hat{g} = \frac{1}{2} \boldsymbol{\tau} : \mathcal{G} : \boldsymbol{\tau}, \quad (21)$$

$$\Phi^g \left(\frac{I_g}{\tau_{eq}(\boldsymbol{\xi}, \vartheta, \epsilon^p)} - 1 \right) = \left(\frac{I_g}{\tau_{eq}} - 1 \right)^{m_g}, \quad (22)$$

where

$$\tau_{eq} = c(\vartheta) \left(1 - (\boldsymbol{\xi} : \boldsymbol{\xi})^{\frac{1}{2}} \right) \ln \frac{1}{(\boldsymbol{\xi} : \boldsymbol{\xi})^{\frac{1}{2}}} \{ 2\kappa_s(\vartheta) - [\kappa_s(\vartheta) - \kappa_0(\vartheta)] F(\xi_0, \boldsymbol{\xi}, \vartheta) \}, \quad (23)$$

$$c(\vartheta) = \text{const.},$$

$$F = \left(\frac{\xi_0}{1-\xi_0} \frac{1 - (\boldsymbol{\xi} : \boldsymbol{\xi})^{\frac{1}{2}}}{(\boldsymbol{\xi} : \boldsymbol{\xi})^{\frac{1}{2}}} \right)^{\frac{2}{3}\delta} + \left(\frac{1 - (\boldsymbol{\xi} : \boldsymbol{\xi})^{\frac{1}{2}}}{1-\xi_0} \right)^{\frac{2}{3}\delta}, \quad (24)$$

and

$$I_g = \bar{b}_1 J_1 + \bar{b}_2 (J_2')^{\frac{1}{2}} + \bar{b}_3 (J_3')^{\frac{1}{3}}. \quad (25)$$

$\bar{b}_i$ ($i = 1, 2, 3$) are the material parameters, J_3' is the third invariant of deviatoric part of the Kirchhoff stress tensor.

Now, taking into account the postulates for microdamage evolution, and assuming that tensor $\mathcal{G}$ can be written as a symmetric part of the fourth order unity tensor $\mathcal{I}$ [20]

$$\mathcal{G} = \mathcal{I}^s, \quad \mathcal{G}_{ijkl} = \frac{1}{2} (\delta_{ik}\delta_{jl} + \delta_{il}\delta_{jk}), \quad (26)$$

we can write the explicit form of the growth function $\hat{g}$ as

$$\hat{g} = \frac{1}{2} (\tau_I^2 + \tau_{II}^2 + \tau_{III}^2). \quad (27)$$

Fig. 9 The comparison of the numerical results for cutting depths 50 μm (left column) and 100 μm (right column) at time instant $3.5 \cdot 10^{-5}$ s for cutting speed 12 ms^{-1}

Fig. 10 The comparison of the numerical results for cutting depths 50 μm (left column) and 100 μm (right column) at time instant $7.0 \cdot 10^{-5}$ s for cutting speed 12 ms^{-1}

Fig. 11 The comparison of the numerical results for cutting depths 50 μm (left column) and 100 μm (right column) at time instant $1.0 \cdot 10^{-4}$ s for cutting speed 12 ms^{-1}

The gradient of $\hat{g}$ with respect to the stress field gives us the following matrix representation of a tensor describing the anisotropic evolution of microdamage

$$\frac{\partial \hat{g}}{\partial \boldsymbol{\tau}} = \begin{bmatrix} g_{11}\tau_I & 0 & 0 \\ 0 & g_{22}\tau_{II} & 0 \\ 0 & 0 & g_{33}\tau_{III} \end{bmatrix}. \quad (28)$$

In (28) $\tau_I, \tau_{II}, \tau_{III}$ are the principal values of Kirchhoff stress tensor.

It can be noted, that the definition of the threshold stress for microcrack growth function τ_{eq} indicates that the growth term in evolution function for microdamage is active only after nucleation whereas before nucleation we have infinite threshold $\lim_{\xi \rightarrow 0} \tau_{eq} = \infty$.

5. For temperature evolution Eq. (10) we consider the following relation

$$\mathbf{k} = \boldsymbol{\tau}. \quad (29)$$

3 ANN approximation

3.1 Number of material parameters—summary

The adiabatic process, as outlined above, in terms of the discussed model requires identification of 28 material parameters [35]. We propose to split them as follows:

- Five parameters related to viscoplastic evolution (including five dimensionless)— $m_{pl}, \beta^*, \beta^{**}, n_1, n_2$,
- Nine parameters related to anisotropic microdamage evolution (including eight dimensionless)— $m_g, c, b_1, b_2, b_3, \xi^{F*}, \xi^{F**}, m_F, ||L\mathbf{v}\xi_c||$,
- Seven parameters shared with viscoplastic and microdamage evolution (including two dimensionless)— $T_m, \kappa_s^*, \kappa_s^{**}, \kappa_0^*, \kappa_0^{**}, \delta^*, \delta^{**}$,
- Four parameters related to thermal evolution (including two dimensionless)— $\chi^*, \chi^{**}, \theta, c_p$,
- Two elasticity parameters— λ, μ , and
- reference density— ρ_{Ref} ,

with all parameters being non negative. In [35] it was shown that without loss of model's generality one can reduce this number to 22. Next, we present further reduction in number of parameters based on ANN approximation. The reduction consists of work hardening–softening and threshold stress for microcrack growth functions which in our opinion do not cause the loss of generality of formulation nor any mathematical consequences. It gives, as mentioned, final amount of a material parameters equal 19 to handle particular metal behaviour.

3.2 Approximation concept

The concept of ANN approximation of work hardening–softening and threshold stress for microcrack growth functions can be summarised as follows.

One can rewrite the work hardening–softening function Eq. (17) as

$$\kappa = \kappa(\epsilon^P, \vartheta, \xi) = \left\{ 1 - \left[\frac{(\xi : \xi)^{\frac{1}{2}}}{\xi_F} \right]^{\beta(\vartheta)} \right\} \{\kappa_s(\vartheta) - [\kappa_s(\vartheta) - \kappa_0(\vartheta)] \exp[-\delta(\vartheta) \epsilon^P]\}, \quad (30)$$

and threshold stress for microcrack growth function Eq. (23) as

$$\tau_{eq} = \tau_{eq}(\epsilon^P, \vartheta, \xi) = 2c(\vartheta) \left(1 - (\xi : \xi)^{\frac{1}{2}} \right) \ln \frac{1}{(\xi : \xi)^{\frac{1}{2}}} \left\{ \kappa_s(\vartheta) - \frac{1}{2} [\kappa_s(\vartheta) - \kappa_0(\vartheta)] F(\xi_0, \xi, \vartheta) \right\}. \quad (31)$$

Now we propose the following approximations for both functions

$$\kappa^* = \kappa(\epsilon^P, \dot{\epsilon}^P, \vartheta, \xi) = \xi_\kappa \mathcal{A}, \quad \text{where} \quad \xi_\kappa = 1 - \left[\frac{(\xi : \xi)^{\frac{1}{2}}}{\xi_F} \right]^{\beta(\vartheta)}, \quad (32)$$

and

$$\tau_{eq}^* = \tau_{eq}(\epsilon^P, \dot{\epsilon}^P, \vartheta, \xi) = \xi_{eq} \mathcal{A}, \quad \text{where} \quad \xi_{eq} = \left(1 - (\xi : \xi)^{\frac{1}{2}} \right) \ln \frac{1}{(\xi : \xi)^{\frac{1}{2}}}, \quad (33)$$

where $\mathcal{A}$ is an ANN approximation which keeps the effects of equivalent plastic strain, strain rates and temperature on flow stress (from experiment), while functions ξ_κ, ξ_{eq} introduce the phenomenological scaling only due to anisotropic microdamage effect. Hence, after ANN learning process, $\mathcal{A}$ it is a function which for known values of $\epsilon^P, \dot{\epsilon}^P, \vartheta$ gives the current value of flow stress in a material point which needs to be scaled by ξ_κ, ξ_{eq} to introduce microdamage effects. It is important to notice, that the structure of Eqs (32) and (33) is in agreement with previously used explicit functions [35] which were obtained from micromechanics considerations at the level of a single void [3].

By eliminating κ_s, κ_0 and δ from material functions in fact we reduce the number of material parameters by 6 cf. Eqs (18)₂, (18)₃, (18)₄.

The comment requires the introduction of strain rate ($\dot{\epsilon}^P$) in both approximated functions. Originally in the Perzyna's formulation functions κ and τ_{eq} are quasi-static while the strain rate effects come from a relaxation time (T_m) which is a function of $\dot{\epsilon}^P$. Such a concept has a deep physical interpretation derived from the analysis of a single crystal and a polycrystal behavior (cf. [29]). Nevertheless, in the concept

presented in this paper it is convenient to introduce a strain rate in κ^* and τ_{eq}^* .

Finally, the assumption in the presented formulation of no initial porosity $\xi_0 = 0$ leads to $\tau_{eq}^* \rightarrow \infty$. For such case, according to Sect. 3.1, we reduce our material parameters by 9 and only 10 parameters for a specific steel remain. In most of the times they can be relatively easy taken from the literature—apart from the data needed for ANN learning (strain–stress curves for versus temperatures and strain rates). Such a case is not considered further in the paper.

The details concerning implementation can be found in e.g. [35, 37]—moreover it must be emphasized that the implementation keeps the Lie objective rate.

4 Numerical examples

4.1 Material model implementation

The discussed material model is implemented in Abaqus/Explicit finite element code by taking advantage of a user subroutine VUMAT, which is coupled with Abaqus system [1]. It's important to mention that Abaqus/Explicit utilizes central-difference time integration rule along with the diagonal (“lumped”) element mass matrices. We use so called element deletion method to remove damaged elements from the mesh [34]—elements in which for every integration point fracture porosity was reached.

4.2 Identification of DH-36 steel parameters

The experimental results are taken from [15]. In [15] the authors investigated the behavior of DH-36 steel for three different strain rates (0.001, 0.1 and 3,000 s^{-1}) and for the sake of comparison for six different temperatures (77, 296, 400, 500, 600 and 800 K). The DH-36 steel belongs to the class of high strength structural low-alloy steels. The DH-36 steel distinguishes itself due to good weldability, toughness, good ductility and plasticity (true strain $> 60\%$) under various temperatures and loading rates. The steel has the characteristics of the bcc structure, hence it belongs to the so called ferritic steels. The mechanical properties of the DH-36 steel are strongly affected by impurities in its internal structure. It is important, that the processing (rolling) of the DH-36 steel, can induce the anisotropy of its structure [8].

In order to finalize the overall concept we must assume that such experimental data set is sufficient to cover all characteristic phenomena for phenomenological description of analyzed steel. However, it should be emphasized, that in our case for modelling of extremely fast thermomechanical processes, the range of strain rates and simultaneously temperatures being analyzed should be increased. It is due to

the fact that it would guarantee that not only the approximation but also extrapolation from ANN will be conducted with acceptable errors.

We used the ANN with single hidden layer (16 neurons) using constrained truncated Newton algorithm for learning. The input consisted of equivalent plastic strain, strain rate and temperature while the target consisted of corresponding flow stress. Both input and output were directly taken from the experimental (isothermal) curves, as provided in [15]. The results of approximation/extrapolation are presented in Figs 2, 3, 4 and 5.

In Fig. 2 we present the original data set (isothermal) taken from [15] and its ANN approximation—please notice, that the approximation is performed with negligible error. Finally, since the analysis of a 3D body under extreme loading conditions leads to a broad range of strains and temperature localizations we need to provide a good extrapolation both in strains and temperature axes as shown in Figs 3 and 4.

Figure 5 representing the extrapolation through the strain rates, requires separate discussion. Due to the fact that the experimental results cover only 0.001, 0.1 and 3,000 s^{-1} rates, the ANN extrapolation can lead to nonphysical answer (cf. plot for 10,000 s^{-1} in Fig. 5). Therefore, during computations we assumed three strain ranges $0 \div 0.001 s^{-1}$, $0.001 \div 1 s^{-1}$ and $> 1 s^{-1}$. In case during the analysis the strain rate was in one of those ranges we induce to take the values from the flow surface for rates 0.001, 0.1 and 3,000 s^{-1} , respectively.

Finally, in Table 1 we present a complete set of parameters, identified in the sense of numerical calibration, for analyzed steel. Note, that material parameters that are reduced are marked within a box in Table 1. Intentionally we additionally omitted three more parameters m_F , $||L\nu\xi_c||$, ξ^{F**} assuming that fracture porosity is constant.

In Fig. 6 the comparison of the strain–stress curves (isothermal and adiabatic) from the experiment and the model for a strain rate 3,000 s^{-1} and the initial temperature 296 K are provided as an example. The adiabatic curve given by the model is obtained from 3D simulation being a model mapping experimental set up [15] while the isothermal one is from ANN.

4.3 Orthogonal cutting analysis

4.3.1 Geometry

The analyzed HSM set up is presented in Fig. 7. The tool is rigid and its geometry is described by an exterior angle equals to 7° and the fillet radius equals to 3 μm . The dimensions of the workpiece (3D) are length 2,000 μm , height 200 μm and out of plane thickness 5 μm .

The boundary conditions on a workpiece are applied at the bottom, front and rear surfaces, while rigid tool can move in

horizontal direction only with constant velocity 12 ms^{-1} . The initial conditions assumed room temperature.

We used 8-node linear brick, reduced integration element (C3D8R) with approximate size $10 \mu\text{m}$.

Friction coefficient between surfaces in contact is assumed to be equal 0.3.

4.3.2 Results

Global analysis

The comparison of the reaction on a tool for cutting depth 50 and $100 \mu\text{m}$ is presented in Fig. 8. It is important to recall, that those results are the most important for machinery designers. We observe that an average force that acts on the tool for cutting depth $50 \mu\text{m}$ is around 0.4 N while for cutting depth $100 \mu\text{m}$ it reaches approximately 0.8 N.

Local analysis

The complete local analysis of orthogonal cutting problem comprises of: creation of shear bands, machined surface roughness, level of plastic strains, local temperature, chip thickness etc. Nevertheless, we regard this numerical example as a benchmark since our analysis are simplified.

The comparison of the HMM stresses, the equivalent viscoplastic strains, the temperature, the porosity maps and privileged damage directions for cutting depths 50 and $100 \mu\text{m}$ at time instant $3.5 \cdot 10^{-5}$, $7.0 \cdot 10^{-5}$, 10^{-4} s are presented in Figs 9, 10 and 11, respectively. Note, that for easier analysis of the maps, each time in rows we use legend with the same upper and lower bounds.

The first look shows severe fragmentation of a chip for cutting depths $50 \mu\text{m}$. Likewise in the experiment (cf. [14]) peak stresses are near tool tip and their spatial distribution properly maps the ability for shear band formation. The equivalent plastic strains can be even 2, while local temperature increase is close to 450 K. The chip is thicker than initial cut depth.

Let us now discuss the results coming from anisotropy of damage which is described by the model. Recall, that we use second order tensor to describe directional behavior of damage (microdamage tensor ξ) and also the physical interpretation of ξ is that the Euclidean norm of the microdamage field defines the scalar quantity called porosity while its principal values indicate damage plane as one perpendicular to maximal principal value of ξ .

In Figs 9, 10 and 11 porosity maps and plots of principal directions of ξ (in selected material points) are presented. Initial value of porosity for metals circulates around $6 \cdot 10^{-4}$ [3, 16, 18] while fracture porosity in our case is equal to 0.3. That is why it is hard to present such quantity on contour maps, hence we use logarithmic scale (see legend for porosity). However porosity does not provide any informa-

tion regarding the damage directions. The situation is different when we analyse principal directions of ξ , remembering that damage plane is one perpendicular to maximal principal value (red arrow on plot). Due to elasto-viscoplastic waves traveling during simulation through the workpiece those are the possible directions of creation new surface changes. However, it is clearly visible that close to the tip of the tool they always show correct direction of macrodamage propagation. Hence using the presented material model, we can trace the direction of softening, and predict damage pattern.

5 Conclusions

In this paper the reduction of material parameters by using ANN approximation for selected material functions plays the key part. The material model chosen for application of the ANN approximation concept is described by recently proposed Perzyna's type viscoplasticity including anisotropic measure of damage [5, 29, 35]. This model is designed for metals under extreme loading conditions and originally introduces 28 material parameters for analysis under adiabatic conditions. In the paper we show that this number of material parameters can be reduced to 19 using the concept of ANN approximation. It is important that the introduced approximation does not violate proper understanding of fundamental phenomena derived from analysis of a single crystal and a polycrystal behavior postulates originally by Perzyna (cf. [28]).

Apart the concept of ANN approximation itself, we are providing the short description of the material model which is used as an example, its implementation in Abaqus/Explicit finite element software and application to HSM problems. Finally we can conclude that ANN approximation is very effective, however considering numerical analysis of high speed phenomena which covers a broad range of temperature (up to melting point), strain rates (up to 10^7 s^{-1}), strains (up to 2) etc. the experimental collection of data must be large. It is due to the fact that the answer from ANN (in particular extrapolated values) depends on data set used for learning.

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