Guest Editors' Foreword

Tamal K. Dey · Steve Y. Oudot

Received: 2 May 2013 / Accepted: 2 May 2013 / Published online: 16 May 2013
© Springer Science+Business Media New York 2013

This special issue is dedicated to a set of selected papers that were presented at the 28th Annual Symposium on Computational Geometry held at Chapel Hill, North Carolina, USA during June 17–20, 2012. The six papers appearing in this issue have been selected based on their rating by the program committee as well as our consideration to promote the young field of computational topology that has brought new enthusiasm enriching the area of discrete and computational geometry. The first two of the papers discussed below have addressed well-known open problems in our field, while the others have addressed some of the key questions arising in the interface of computing with geometry and topology. We hope that the readers find this issue useful.

The paper by Natan Rubin tackles a long-standing problem in our field and makes significant progress. Given a set P of n points moving along curves in the plane, the goal is to estimate the total number of combinatorial changes that occur in the Delaunay triangulation of P during the motion. The most famous instance of this problem is when the points undergo a uniform motion at unit speed along lines. The best known lower bound for this instance is quadratic in n , while the best known upper bound is cubic in n . This paper reduces the upper bound to a near-optimal $O(n^{2+\varepsilon})$, under a somewhat restrictive but purely topological motion model where any four points of P can be co-circular at most twice, and either every ordered triple of points of P is collinear at most once or every unordered triple of points is collinear at most twice.

T. K. Dey (✉)
The Ohio State University, Columbus, OH 43210, USA
e-mail: dey.8@osu.edu

S. Y. Oudot
Inria Saclay – Île-de-France, 1 rue Honoré d'Estienne d'Orves, 91120 Palaiseau, France
e-mail: steve.oudot@inria.fr

The paper by Ábrego et al. considers a well-known open problem in graph theory. Given a complete n -vertex graph K_n , what is the minimum number of crossings among edges needed to draw K_n in the plane. This number, known as the crossing number of K_n , has a well-known upper bound $Z(n)$ which is conjectured to be the right bound. A modified version of the problem called 2-page drawing crossing number of K_n asks to draw all vertices on a line and all edges in the two half-planes bounded by the line. This version of the problem has also been well studied and is conjectured to have $Z(n)$ as the right bound. The paper of Ábrego settles this conjecture by proving that the bound is indeed $Z(n)$.

The paper by Don Sheehy addresses the problem of approximating persistence diagrams of Vietoris–Rips filtrations with a sparsification scheme. It is well known by now that the Vietoris–Rips complex and its filtration capture topological information of a space if its vertex set samples the space sufficiently densely. As a clique-completion of a neighborhood graph, the Vietoris–Rips complex can be huge in size, thereby stifling further computations in practice. Sheehy shows how the so-called hierarchical net-trees can be used to systematically prune the vertex set of a Vietoris–Rips filtration so as to generate a sparser filtration of controlled size which still approximates the persistence diagram of the original. Sparsifying filtrations while preserving their topological structure is a key step toward applications in topological data analysis. To this end, leveraging concepts and techniques used for computing proximities in metric spaces is a novel and elegant idea.

The paper by Herbert Edelsbrunner, Brittany Terese Fasy and Günter Rote reveals some surprising facts about mixtures of Gaussians in high dimensions. It is known that the sum of n Gaussians can have fewer or more than n modes, however little is known about their exact number and the conditions under which extra modes may appear. Considering a particular canonical case where n unit Gaussians are placed at the vertices of a scaled version of the standard $(n - 1)$ -simplex, the paper shows that there is a range of values of the scaling factor, whose width grows like $\sqrt{n}$, throughout which the number of modes is exactly $n + 1$ as an additional mode appears at the barycenter of the simplex. Thus, not only can the appearance of such extra modes be predicted, but it also becomes more likely as the dimension increases. The paper provides further evidence by showing that the total number of modes may increase super-linearly in the number of Gaussians.

The paper by Jeff Erickson and Amir Nayyeri proposes a unifying framework for the processing of normal curves on triangulated surfaces. Given such a curve c , the main idea is to replace the surface triangulation by a so-called *street complex*, made of pieces of constant description complexity, whose advantage is to represent the same surface while reducing the size of the description of c . Tracing c within the street complex thus takes polynomial time, as does the translation from the triangulation to the street complex and vice-versa. The beauty of the approach resides in its unifying and simplifying view of several fundamental topological questions about normal curves, such as determining whether such a curve is contractible, which were solved previously using very different and significantly more involved techniques.

The paper by Salman Parsa presents a deterministic $O(n \log n)$ time algorithm for computing the Reeb graph of a scalar function defined on a simplicial complex. In recent years, efficient computation of Reeb graphs has attracted a lot of attention

because of its variety of applications. Earlier a randomized algorithm with the average case time complexity of $O(n \log n)$ was known. Parsa's algorithm settles the question of computing the Reeb graphs deterministically in optimal time.

Acknowledgments The guest editors wish to thank the anonymous referees for their careful reading of the papers and for their detailed reviews. They also thank the authors for their great efforts toward improving their papers and taking the reviewers' comments into account.