Guest Editors' Foreword

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This special Issue of Discrete & Computational Geometry is dedicated to the presentation of some recent results in the area of geometric transversals and Helly-type theorems. The idea of the special issue grew out of the highly successful workshop, called Transversal and Helly-type Theorems in Geometry, Combinatorics, and Topology, which took place at the Banff International Research Station (BIRS) between the 20th and the 25th of September, 2009. This issue contains 12 articles, and all of them were refereed according to the usual high standards of Discrete & Computational Geometry.

Helly's theorem is perhaps one of the most cited theorems in discrete geometry and has stimulated numerous generalizations and variants. The search for Helly numbers in different "universes" other than convex sets has inspired many articles; for instance in this volume, J.L. Arocha and J. Bracho show that the lattice of linear partitions in a projective geometry of rank n has a Helly number.

There are many interesting connections between Helly's theorem and its relatives, the theorems of Radon, of Carathéodory, and of Tverberg. In fact, one of the most beautiful theorems in combinatorial convexity is Tverberg's theorem, which is the r -partite version of Radon's theorem, and it is very closely connected to the multiplied, or colorful, versions of the theorems of Helly, Hadwiger, and Carathéodory. In this spirit, J. Bokowski, J. Bracho and R. Strausz present some generalization of the Bárány–Carathéodory theorem to oriented matroids of Euclidean dimension 3. R. Strausz introduces the notion of hyperseparoids as a generalization of separoids, and proves a representation theorem for hyperseparoids in terms of Tverberg partitions.

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L. Montejano and D. Oliveros generalize every Helly-type theorem by relaxing the assumptions and conclusions, allowing a bounded number of exceptional sets or points, by introducing the notion of tolerance. They present several results concerning the tolerance versions of the classical theorems of Helly, Carathéodory, and Tverberg.

An important generalization of Helly's theorem is the (p, q) problem introduced first by Hadwiger and Debrunner. On this topic, L. Montejano and P. Soberon give bounds to the piercing number for certain balanced and nonbalanced families of convex sets with the $(p, q)_r$ -property.

One of the problems that has received attention in this field is the problem of finding a line transversal to a family of mutually disjoint compact convex sets. On this subject, A. Heppes presents a paper about super-disjoint $T(3)$ -families of translations of an oval in the plane. Define $\lambda(K, k)$ as the smallest number satisfying the following: if F is a $T(k)$ -family of translates of K , then all members of $\lambda(K, k)F$ have a common transversal line. J. Jeronimo and E. Roldan-Pensado present some bounds on $\lambda(K, 4)$ and $\lambda(K, 3)$, for families of translated copies of a convex body K in the plane.

X. Goaoc, S. König, and S. Petitjean present a paper about isolated line transversals to finite families of (possibly intersecting) balls in $\mathbf{R}^3$ with certain properties, as well as some generalizations to families of semialgebraic ovaloids, whereas B. Aronov, O. Cheong, X. Goaoc, and G. Rote present a paper about isolated line transversals to convex polytopes in $\mathbf{R}^3$.

An application of algebraic topology to discrete geometry is also presented in this special volume. R. Karasev proves a dual Tverberg theorem for hyperplanes based on the following notion of separation: let F_i be a family of $d + 1$ hyperplanes in $\mathbf{R}^d$ in general position, $i = 1, \dots, n$. We say that the n families are separated if the intersection of the n simplices generated by each one of the families is empty. The dual Tverberg theorem presented in this paper is that if n is a prime power and F is a family of $(d + 1)n$ hyperplanes in general position in $\mathbf{R}^d$, then F can be partitioned into n subfamilies which are nonseparated.

J. Ferté, V. Pilaud, and M. Pocchiola describe an incremental algorithm to enumerate the isomorphism classes of double pseudoline arrangements. Counting results derived from an implementation of this algorithm are also presented in this paper.

A. Deza, T. Stephen, and F. Xie show that any point in the convex hull of each of $(d + 1)$ sets of $(d + 1)$ points in general position in $\mathbf{R}^d$ is contained in at least $\lceil (d + 1)^2/2 \rceil$ simplices with one vertex from each set.

We hope you enjoy this special volume and we thank all the authors and referees for their work, which was impressive both by its high scientific value and its timeliness.

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