Guest Editor's Foreword

Jeff Erickson

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This special issue of *Discrete & Computational Geometry* consists of five selected papers from the 23rd Annual Symposium on Computational Geometry, held June 6–8, 2007, in Gyeongju, South Korea. These papers cover a broad range of topics and display a rich variety of algebraic, algorithmic, combinatorial, geometric, and topological techniques.

Peyman Afshani and Timothy Chan describe a randomized data structure for approximate halfspace range counting, which uses $O(n)$ expected space and answers queries in $O(\log(n/k))$ expected time, where k is the output. Both the space and query time bounds are optimal in the algebraic decision tree model. A dual formulation of the approximate halfspace range counting problem is to approximate the depth of a point in an arrangement of planes; using similar techniques, Afshani and Chan also derive new algorithms for several related depth problems.

Thorsten Bernholt, Friedrich Eisenbrand, and Thomas Hofmeister describe linear- or near-linear-time algorithms for a wide variety of interval optimization problems that arise in computational biology, by casting them into a common geometric framework. Their main geometric result is a linear-time algorithm for the following natural problem: Given two planar point sets P and Q and a halfplane h , compute the convex hull of the restricted Minkowski sum $(P + Q) \cap h$.

Jean-Daniel Boissonnat, Leo Guibas, and Steve Oudot describe a new multi-scale manifold reconstruction algorithm using witness complexes. Given a finite set of points on an unknown smooth manifold S embedded in some Euclidean space, their algorithm constructs a spectrum of simplicial complexes approximating S at various length scales. The authors first show that a naïve application of witness complexes may not provide correct reconstructions of manifolds of dimension 3 or higher,

J. Erickson (✉)
Department of Computer Science, University of Illinois, Urbana, IL 61801, USA
e-mail: jeffe@cs.uiuc.edu

even for very well-behaved samples, thanks to certain poorly-shaped simplices called slivers. The authors then restore the necessary structural properties by considering a weighted version of the witness complex and replacing the witness points with small witness balls. The reconstruction algorithm combines a farthest-point sampling strategy with a “vertex-pumping” strategy originally developed by other authors to remove slivers from Delaunay triangulations.

Gunnar Carlsson and Afra Zomorodian consider a multidimensional generalization of the now-standard theory of persistent homology. Carlsson, Zomorodian, and others previously showed that the persistent homology of a filtration—a nested sequence of topological spaces—is completely described by a multiset of real intervals, in which longer intervals correspond to persistent topological features and shorter intervals to topological noise. Here, Carlsson and Zomorodian prove that the persistent homology of multidimensional filtrations has a significantly richer algebraic structure, which forbids a similar discrete description. They also propose a natural and easily computable *rank invariant* that identifies many persistent features.

Hazel Everett, Daniel Lazard, Sylvain Lazard, and Mohab Safey El Din given the first complete characterization of Voronoi diagram of three lines in $\mathbb{R}^3$ in general position. In particular, the topology of the Voronoi diagram is identical for any three generic lines; the 1-skeleton consists of all four branches, all of which are unbounded, of a nonsingular curve of degree 4. This characterization relies on a careful study of the algebraic variety structure of this trisector curve, which ultimately boils down to proving that a certain real polynomial of degree 18 in 5 variables (!) is never negative. Building on their characterization, Everett et al. also describe simple algorithms for several primitive operations required to compute Voronoi diagrams of larger sets of lines.

I invited two additional papers to this special issue, which were already far along in the journal review process: “There Are not Too Many Magic Configurations” by Eyal Ackerman, Kevin Buchin, Christian Knauer, Rom Pinchasi, and Günter Rote; and “Line Transversals to Disjoint Balls” by Ciprian Borcea, Xavier Goaoc, and Sylvain Petitjean. Both papers appear in the 20th anniversary special issue of *Discrete & Computational Geometry* (Volume 39, Numbers 1–3).

All the papers in this special issue were reviewed through the normal refereeing process, according to this journal’s usual high standards. I would like to thank the authors and reviewers for all the hard work that made this special issue possible.

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