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On new effects of wheel-rail interaction

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Abstract The paper is devoted to the experimental, theoretical analysis and computer simulation of influence of elastic properties of contact stiffness and wheel-plate stiffness on the forces of vehicle-track interaction. Three types of wheels are considered with different contact stiffness and wheel-plate design. Exemplary simulation of freight car interaction with track which possesses one corrugated rail for each type of wheel is presented.

Keywords Contact stiffness · Rail corrugation · Elastic wheel · Wheel-rail interaction · Simulation

1 Introduction

The cost-efficiency, the maintenance, and the safety of railway operation depend strongly on the quality of the wheelsets-track interaction. From here, the need for the definition of the running behavior of regular vehicles arises, because the prediction of running behavior depends on the determination of the geometrical wheelset-track interaction parameters. Also the environmental impact (vibration, noise) and the recurrent maintenance needs of pavement, sleepers, and rails depend strongly on the surpassing traffic, causing interaction between vehicle and track. The time-dependent variation of the traveling contact forces between the wheelsets and the rails is of highest importance as an origin of load and excitation of the track. The contact problems of wheel-rail interaction in the majorities of studies are limited to the dynamic effects caused by the vertical motion of the contact points under assumption that the contact stiffness is linearly dependent on the load. The studies are conducted taking that the speed of loading is constant and equal to the velocity of the train motion, [1]. In the reality, the contact phenomenon is much more complicated. In particular, the path of the geometrical contact area is more complicated and the horizontal component of velocity of unilateral contact is of oscillatory nature what is discussed in [2–4]. Systems with unilateral contact are very sensitive to errors in values of times at which the structure of the system changes due to loss of contact and due to impacts. Therefore, the simulation procedure ought to be precise [2, 5, 6]. The results of experimental study of two-dimensional contact problem with influence of the wheel-plate and the rim deformation on contact geometry seem to be very important and will be presented in this paper. Additionally, parameters of the contact stiffness and wheel-plate stiffness obtained experimentally will be used for simulation of dynamic contact forces and the wear intensity, that is, the speed at which the wear surface proceeds inwards is a function of the power dissipated by friction in the contact region. The aim of this paper is to evaluate real stiffness parameters of different wheel types being in

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service and to devise a relatively simple model for the simulation of the dynamic interaction of the system bogie-track with more realistic contact parameters of the described process as in previous papers [1,3,6–8].

2 Measurement of wheel-plate stiffness and contact stiffness

It is well known that kinematic excitations caused by wavy pattern on the rails surfaces can lead to strong dynamic interaction. As we can see in the paper [1], taking into account vertical acceleration only it is pointed out that at relatively low running speed $v_o = 50$ km/h resonance vibration occur which can lead to the bouncing between the wheel and the rail. The trajectory of such vibration and change of vertical force for corrugation amplitude $A = 0.010$ mm and wave length $\lambda = 50$ mm are visible in Fig. 1. In this investigation, the wheel-plate was assumed as rigid. The vanishing of the contact force which can be seen on the right side of Fig. 1 indicates that the wheel lifts off the rail.

The change of load in such a range induces changes of the stiffness parameters which are not considered up to now in papers devoted to the dynamic wheel-rail interaction, that is [5,6]. In the very beginning, let us consider results of measurement of wheel stiffness, wheel-plate stiffness, and wheel-rail contact stiffness obtained in the range 0–100 kN done on the special test stand shown in the Fig. 2a. The scheme for the measurement of the displacements is explained on Figs. 2b and 3. Here, the indices 1 and 2 denote the preloaded and the loaded state, respectively (Fig. 2b).

The curves showing the results of measurement on the left and right side of the wheel are given in Figs. 4, 5, and 6. The negative displacement on the side of the flange is caused due to loss of symmetry of the rim cross-section and non-symmetric wheel-plate. The vertical load induces an increase in the bending moment acting on the rim and thereby changes the location of the contact area.

The stiffness parameters depend on the design of the wheel-plate, the shape of the rim, and the manufacturing technology. Full scale experimental investigations concerning the comparison of wear or practically important parameters describing the dynamic interaction of diverse wheel types and rails are difficult and for this reason are conducted very rarely. In this research, measurements on test stand using full scale wheels have been carried out at first. Subsequently, the theoretical analysis and simulation will be conducted. For the

Fig. 1 Shape of corrugation (left 2), relative vertical motion of the wheel center and resonance vibration(1) (left 1) and vertical contact force (right)

Fig. 2 a Special test stand EMS 60 for investigation wheel-rail parameters. b Scheme of measurements

Fig. 3 Laser measurement of wheel-plate and wheel-rail contact stiffness in the test stand

Fig. 4 Results of tired wheel stiffness measurements. Load versus displacement and nonlinear approximation of characteristics

detailed study of parameters values, three type of wheels are taken into account: forged-rolled monobloc wheel made of ER7 material, tired wheel type used in Poland, and pressure poured cast wheels manufactured by Amsted Rail from ER7 material. The choice was connected with the different plate shape, of the rim design and of the manufacturing technique. Therefore, different stiffness parameters can be expected. For fifteen wheels, the wheel-plate stiffness and contact stiffness measurement have been conducted. Exemplary results for three of above measurements are shown in the Figs. 4, 5, and 6. The case of tired wheel is shown in the Fig. 4, while the results of the wheel stiffness measurements for forged/rolled monobloc wheel per UIC of ER7 material are shown on the Fig. 5. The results of the wheel stiffness and the contact stiffness measurements for the pressure poured cast wheels of Amsted Rail from ER7 material are illustrated in Figs. 6 and 7. In Figs. 4, 5, 6, and 7, the colors of the curves generally refer to the colors indicating the displacement in Fig. 2b, that is, the red and the blue curves indicate the displacements at the flange side and the side opposite the flange, respectively. The estimation of measurements errors is also given. It can be seen that the displacements on the both sides of the rim strongly differ from each other. It has to be underlined that the difference is qualitatively significant and that it is dependent on the wheel-plate shape and on the wheel-plate stiffness. As we can state on basis of measurements for all types of wheels, the displacement on left and right side of the rim has opposite directions. Such a case is caused by asymmetrical shapes of the rim and the asymmetrical shape of plate, which induce a

Fig. 5 Results of the forged/rolled monobloc ER7 wheel stiffness measurements. Load versus displacement and linear approximation of characteristics

Fig. 6 Results of cast wheel stiffness measurements (ER7). Load versus displacement measured on both sides of the wheel and nonlinear approximated characteristics

rim rotation in the plane perpendicular to the rim axis. As a consequence, changes of wheel-rail contact area occur. We will discuss this problem for various kind of wheels later.

The results for measurements of parameters for the wheel types described above are given in the following figures. As mentioned we distinguish between the stiffness of the wheel-plate c_p , the contact stiffness c_c , and total stiffness of the wheel c_w . Using linearized stiffnesses, the relation between c_p , c_w and c_c is given by:

$$1/c_w = 1/c_c + 1/c_p \quad (2.1)$$

Characteristics of various wheels stiffness are obtained using linear and nonlinear approximations.

On Fig. 4, we can see two characteristics of the tyred wheel stiffness. With the increment of vertical load, an increase in displacement R_R on the side opposite to the flange in the direction toward the center of wheel

Fig. 7 Results of cast wheel ER7 contact stiffness. Load versus displacement measured on both sides of the wheel rim

is observed. This displacement is shown on the right part of the graph. On the flange side of the wheel, we can see displacements in the opposite direction, that is, to the outside, the distance to the wheel center is increasing. The results of measurements are shown on the left part of the Fig. 4. The characteristics for the case of monobloc forged-rolled wheel are shown in Fig. 5. In this case, linear approximation is used.

The characteristics of pressure poured cast wheel ER7 measurements are shown in Fig. 6. Load versus relative displacement between wheel center and rim measured on both sides of the wheel is approximated in nonlinear way. From the comparison of the results described above, it can be seen that the forged-rolled tyred wheel has much greater rotational flexibility of rim than the monobloc forged-rolled wheel and the cast wheel of ER7 material.

The angle of rim rotation α is denoted by the following formula:

$$\operatorname{tg} \alpha = (\Delta R_R - \Delta R_L) / b \quad (2.2)$$

where ΔR_R and ΔR_L denote the increases of displacement on the right and left side, respectively, and b is the rim width.

In a similar way, characteristics of the contact stiffness and the stiffness of the wheel-plate were obtained for the pressure poured cast wheel ER7. Results of the approximation are shown in Figs. 7 and 8. The rim rotation, which is proportional to the differences of displacements on both rim sides, is smaller in the case of the forged-rolled monobloc ER7 wheel than in the case of the tyred wheel. This is visible also on results shown in Fig. 9.

The advantages of various pressure poured cast wheels manufactured in North America in technically advanced, automated way, including also wheels of Class B and C, are discussed in Ref. [9].

The safety of railway operation depends on the quality of the wheelset's interaction with the track (rails). Without determination of the geometrical parameters as function of load, the prediction of the rolling behavior seems to be properly not possible. For this reason, the investigations described above are necessary.

The linear approximated wheel stiffness, the plate stiffness, and the contact stiffness for a load of 100 kN, which are obtained from the average value of measurements on both sides, are given for all types of wheel in the Table 1.

The results for the measurements of displacements on both sides of the wheel are qualitatively different because of the asymmetry of the cross-section of rim and plate. For the above-mentioned three types of wheels, the measured values are given in the Table 2.

The nonlinear characteristics of wheel-plates and characteristics of rim-rail contact load-displacement are shown in Figs. 8 and 9, respectively.

Fig. 8 Results of the plate stiffness measurements (average displacement value from both sides from both sides) for different wheel types

Fig. 9 Results of the contact stiffness measurements (average value of the displacement from both sides) for different wheel types

Table 1 Results of the stiffness measurement on the test stand (wheel-plate, contact, wheel-rail)

Wheel type	Stiffness		
	Wheel-plate (MN/m)	Rim-rail contact (MN/m)	Wheel-rail (MN/m)
Tyred wheel	500	500	250
Forged-rolled monobloc ER7	910	579	354
Cast wheel Griffin ER7	580	505	270

3 Influence of rim rotation on the contact geometry

As follows from the presented measurement results, the stiffness characteristics are strongly dependent on the applied load. In a similar way, the angle of rim rotation is a function of the load. The value of the above-mentioned rotation strongly depends on the design of the wheel, which follows from the comparison of the

Table 2 Exemplary results of displacement of axle wheel by vertical load 100 kN

Wheel type	Displacement (mm)	max (mm)	min (mm)
Tyred wheel			
Opposite side of flange	0.68 ± 0.03	0.65	0.71
Side of flange	-0.26 ± 0.03	-0.29	-0.23
Forged-rolled monobloc ER7			
Opposite side of flange	0.48 ± 0.03	0.45	0.51
Side of flange	-0.21 ± 0.03	-0.24	-0.18
Cast wheel Griffin ER7			
Opposite side of flange	0.54 ± 0.03	0.57	0.51
Side of flange	-0.19 ± 0.03	-0.22	-0.16

Fig. 10 Results of the wheel stiffness measurements (average displacement value from both sides from both sides) for different wheel types **Fig. 11** Position of the contact points geometry for three angle values of wheel-rail rotation (S1002 and UIC 60 E1 profiles)

displacements shown in Figs. 4, 5, and 6. The differences of the displacement on the left and the right side of the rim are significant, especially in the case of the tyred wheel. The rotation for the cast wheel ER7 which has a parabolic shape of the plate (Fig. 6) is moderate (Fig. 10). As a consequence of the relative rotation between the profiles of wheel and rail, changes of the position of the contact points occur [10, 11]. The position of the contact points for the relative angle of : $+3^\circ$, 0 , -3° in the case of S 1002 wheel profile and rail UIC 60 E1 is shown in Fig. 11. Here, the displacement between wheel and rail is varied in the range of about 0.04 m.

The configuration of the contact points between wheel rim and rail of -2° on the left wheel and of $+2^\circ$ on the right wheel is displayed in Fig. 12. Here, the rail inclination of 1/40 is taken into account. The measurements and analysis presented above were done assuming quasi-static loading and neglecting any changes of roll angle

Fig. 12 Position of the contact points of wheel set for the case of wheel-rail rotation angle: -2° , $+2^\circ$ and rail inclination (1/40)

of the wheelset. In reality, dynamical wheel-rail interaction takes place, similar as shown in Figs. 1 and 14. Such a dynamical process with relatively high frequency requires the consideration of wheel dynamics [12] and rail dynamics [13] using a traveling wave approach. For a case of load moving with fast varying speed, the simulation is much more complicated as that presented in our analysis without analytical solution.

In a simplified model of the wheel [12], the rim is modeled as a beam having the bending stiffness EI , cross-section A , moment of inertia I , and mass density ρ . The wheel-plate is represented by an elastic foundation having the stiffness parameter q and damping parameter η . Using the shear force Q and the contact force $F_y(x, t)$, the motion for the displacement w and rotation Ψ can be written as:

$$\begin{aligned} \frac{\partial Q}{\partial x} + F_y(t, x) - qw - \eta \frac{\partial w}{\partial t} - \rho A \frac{\partial^2 w}{\partial t^2} &= 0, \\ EI \frac{\partial^2 \Psi}{\partial x^2} + Q - \rho I \frac{\partial^2 \Psi}{\partial t^2} &= 0 \end{aligned} \quad (3.1)$$

By setting the rotation equal to the slope of displacement, that is, $\Psi = (\partial w / \partial x)$ and eliminating the shear force Q , the following equation of motion describing a Rayleigh beam is obtained:

$$EI \frac{\partial^4 w}{\partial x^4} + \rho A \frac{\partial^2 w}{\partial t^2} - \rho I \frac{\partial^4 w}{\partial t^2 \partial x^2} + qw + \eta \frac{\partial w}{\partial t} = F_y(t, x). \quad (3.2)$$

The motion of the rail is described by the following equation:

$$EI_b \frac{\partial^4 w}{\partial x^4} + T \frac{\partial^2 w}{\partial x^2} - m \frac{\partial^4 w}{\partial t^2} + h \frac{\partial w}{\partial t} + cw = F_y(t, x)$$

where E , I_b , T , m , h and c denote the bending stiffness of the rail, the reference longitudinal force, the mass density of the beam, damping and stiffness per length of the support, respectively. For the contact force $F_y(x, t)$, a harmonic force is assumed, which moves with fluctuating velocity.

Here, F_0 and F_1 denote the reference value and oscillation amplitude of the force. The angular frequency of the oscillation is given by ω_1 , δ is the Dirac Function, x is the longitudinal coordinate, V denote speed of motion, ε is relative amplitude of the oscillation, which has the frequency $k_0 V$, where k_0 is the wave number of corrugation).

A further investigation of the new phenomena which is based on the presented modeling of the wheel rim and the rails as beams will be presented in a separate paper.

4 Computer simulation of freight car-track interaction for the case of one corrugated rail

In the majority of papers devoted to wheel-rail contact problems, the main assumption in the modeling is that the bodies are infinitely rigid, only a finite contact compliance of rolling surfaces is taken into account. The dynamical behavior of a vehicle-track system depends on the parameters of the wheel-rail system, on the wheel-rail contact geometry and on the dynamical properties of the adjoining structures. As a simple example, a freight car bogie shown in Fig. 13 with flexible wheels, which is rolling along straight track possessing one corrugated rail, is considered.

The corrugation of the rail is described by a vertical disturbance given by the following equation:

$$Z = A (1 - \cos(kx)), \quad k = 2\pi/\lambda, \quad (4.1)$$

where A and λ denote the amplitude and the wavelength of the corrugation.

Fig. 13 Scheme of the bogie used in dynamic simulation of the freight car

Fig. 14 Response of the bogie's front wheel interacting with a corrugated rail (amplitude $A = 0.03$ mm) at a speed of $v_0 = 50$ km/h

The equations of motion for the simulation are assumed to have the following form:

$$\mathbf{M}\ddot{\mathbf{u}} + (\mathbf{B}_s + \mathbf{B}_a)\dot{\mathbf{u}} + (\mathbf{C}_s + \mathbf{C}_a)\mathbf{u} - \mathbf{F}(\mathbf{u}, \dot{\mathbf{u}}) = \mathbf{0} \quad (4.2)$$

where the matrices $\mathbf{B}_s$ and $\mathbf{C}_s$ are symmetric and the properties of the skew-symmetric matrices are : $\mathbf{B}_a = -\mathbf{B}_a^T$, $\mathbf{C}_a = -\mathbf{C}_a^T$. The vector of forces $\mathbf{F}(\mathbf{u}, \dot{\mathbf{u}})$ contains nonlinear terms in the generalized coordinates. The formulation of equation in symbolic way used for simulation is:

$$\mathbf{M}\ddot{\mathbf{u}} + \Phi_{\mathbf{u}}^T \lambda - \mathbf{L}^T \mathbf{F}(\ddot{\mathbf{u}}, \mathbf{u}) = \mathbf{0}, \quad \Phi(\mathbf{u}, t) = \mathbf{0}, \quad (4.3)$$

where $\mathbf{M}$ is the matrix of inertia, $\mathbf{u}$ is displacement vector depending on time, Φ is the matrix of constraints, $\Phi_{\mathbf{u}}$ is the matrix of constraints gradient, $\mathbf{L}$ is the projection matrix of forces on direction of displacements $\mathbf{u}$, $\mathbf{F}(\mathbf{u}, \dot{\mathbf{u}})$ is the matrix of forces, external forces and inertial forces depending on rotational speed (centrifugal and Coriolis terms). Let us describe the constraints expressed by the matrix $\Phi(\mathbf{u}, t)$ in the following modified way by introducing the new matrix Ψ

$$(\mathbf{K} - \omega^2 \mathbf{M})\mathbf{u} = \mathbf{0}, \quad \Psi(\mathbf{u}, \mathbf{v}, \lambda) = \mathbf{0}, \quad \dot{\mathbf{u}} - \mathbf{v} = \mathbf{0}, \quad (4.4)$$

as the forces depend on displacements and velocities (non-classic constraints). Then we can apply a g-stiff algorithm elaborated by Gear [14].

The behavior of a freight car running on a track with one corrugated rail is simulated. A bogie of the freight car equipped with two wheelsets, each one consisting of an axle and two wheels of given type with profiles S 1002 interacting with typical European track with rails UIC 60 E1 is performed. The vertical disturbance of the corrugated rail's rolling surface is described by Eq. (4.1) using an amplitude of $A = 0.030$ mm and a wave length of $\lambda = 100$ mm. The length of the corrugated section of the rail is $\Delta l = 1,000$ mm. For the wheel-rail friction coefficient, a value of 0.4 is assumed.

Vehicle model parameters are shown in Tables 3 and 4.

Table 3 Mass property of model

Vehicle element	M (kg)	I_{xx} (kg m ²)	I_{yy} (kg m ²)	I_{zz} = kg m ²
Carbody	70,000	48,000	520,000	50,000
Bogie	2,300	1,975	1,460	2,750
Wheel set	1,300	690	100	690
Axle box	30	10	10	10

Bogie geometry: wheel set tape circle distance—1.5 (m), rolling radius—0.42 (m), Bogie wheelbase—1.80 (m)

Table 4 Suspension properties

Vehicle element	K_x (N/m)	K_y (N/m)	K_z (N/m)
Primary suspension spring	650000.0	650000.0	450000.0
Lateral primary bumpstop	0.0	7.5	9.5
Secondary suspension—center pivot	6000000.0	6000000.0	6000000.0

For the traveling speed, few values from the range (20–100 km/h) are assumed. The case for $v_0 = 50$ km/h is shown on Fig. 14. Evaluation of forces is studied using modified “Simpack” program in our laboratory. Normal contact vibration and impacts excited by corrugation on the rolling surfaces of one rail are of interest. It follows from our numerical analysis that even in the case of the assumed amplitude of corrugation, wave length and speed loss of wheel-rail contact occur for all types of wheels. The above approach of modeling can be treated as generalization of investigations considered in [1, 13, 15].

5 Conclusion

The wheel-plate stiffness and the elastic properties of the wheel-rail contact were determined by experimental measurement for three types of wheels (forged-rolled monobloc wheel per UIC of ER7 material and tyred wheel type used in Poland and pressure poured cast wheels manufactured by Amsted Rail from ER7 material). A rotation of the wheel rim due to the loading by wheel-rail by vertical forces is confirmed, which influences the wheel-rail contact geometry considerably. As an example, a freight car moving along straight track possessing only one corrugated rail was considered. The simulation of the system with only one corrugated rail is a generalization of the study considered in [13]. The influences of the wheel-plate shape and the contact stiffness on contact forces are demonstrated. The effects discussed above supplement the effect of flexible wheelsets and rails studied in Ref. [15–17].

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