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Temperature and thermal stresses in material of a pad during braking

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Abstract The transient temperature field and corresponding quasi-static thermal stresses are analysed in a system consisting of a semi-space and a strip. The strip is heated on its outer surface by a heat flux with the intensity equal to the specific power of friction during braking with a uniform retardation. The evolution and distribution in depth from a surface of friction for temperatures and thermal stresses were investigated for the metal-ceramic FMK-11 material of the strip.

Keywords Braking · Friction · Heat generation · Temperature · Thermal stresses

Nomenclature

A_{a1}	Nominal friction surface of a pad, m^2
A_{a2}	Nominal friction surface of a disc, m^2
$A_{ov} = A_{a1}/A_{a2}$	Overlap coefficient, dimensionless
d	Strip thickness, m
E	Young's modulus, Pa
f	Coefficient of friction, dimensionless
$H()$	Heaviside's step function
K	Heat conductivity, $\text{W}/(\text{mK})$
k	Thermal diffusivity, m^2/s
p_0	Pressure, Pa
$q_0 = \gamma f p_0 V_0$	Intensity of the frictional heat flux, W/m^2
T	Temperature, $^{\circ}\text{C}$
$T_{\max}$	Maximal temperature, $^{\circ}\text{C}$
$T_0 = q_0 d / K_p$	Temperature scaling factor, $^{\circ}\text{C}$
T_a	Surrounding ambient temperature, $^{\circ}\text{C}$
t	Time, s
t_s	Braking time, s
t_c	Time of the sign change of lateral stress, s

$t_{\max}$	Time, when maximal temperature is reached, s
$u = 2\sqrt{\tau}$	Dimensionless
$u_s = 2\sqrt{\tau_s}$	Dimensionless
V	velocity sliding, m/s
V_0	initial velocity sliding, m/s
x, y, z	Spatial coordinates, m

Greek symbols

α	Linear thermal expansion coefficient, K^{-1}
γ	Heat participation factor, dimensionless
ν	Poisson's ratio
$\sigma_0 = \alpha E T_0 / (1 - \nu)$	Stress scaling factor, Pa
σ_x, σ_y	Normal stresses, Pa
$\sigma_{x,y}^* = \sigma_{x,y} / \sigma_0$	Dimensionless

Indexes

c	Calliper (foundation)
d	Disc
p	Pad (strip)

1 Introduction

Basic elements of disc brakes are a cast-iron disc, which rotates with the wheel, friction material (brake pads) and a calliper fixed to the steering knuckle. When the brakes are applied, hydraulically actuated pistons move the friction pad into the contact position with the rotating disc. Due to friction between surfaces of the pad and the disc, the kinetic energy of the rotating wheel is converted into heat, by which vehicle is to stop after a certain distance.

Mathematical and numerical modelling of the temperature and the thermal stresses is the important problem at a design stage of brake systems [1–3]. “The heat problems of friction” refer to problems of heat conductivity and calculation of the temperatures in friction couples during braking. Comparison of temperature values obtained from analytical solutions to heat problems of friction, with the known results of experimental investigations of temperature regimes of the brakes, showed that for the calculation of the temperature, the solution to the one-dimensional transient heat conduction problem can be used [4–6].

The heat problems of friction are considered in two approaches. The first variant consists in the simultaneous solution to the heat conduction equations for both pad and the disc with the subsequent determination of heat flux intensities on the surface of friction [7–12].

In the second variant, the elements of friction pair are divided in thoughts, and then on each of friction surfaces, the intensity of heat fluxes are set in such a way that their sum equals the specific power of friction [13–15]. For this purpose, the heat participation factor is usually introduced experimentally or with the use of empirical formulas [16, 17]. Having taken the above assumptions into account, the solutions to heat conduction problem of friction during braking for a homogeneous or a composite strip have been obtained [18, 19]. The thermal stresses caused by a uniform heat pulse heating the outer surface of a strip were investigated in paper [20].

Damage to materials at thermal cracking takes place due to a field of tensile stresses formed during local heating of contact surface. The cracks, oriented along the direction of sliding, may be generated when the stresses on heated surface exceed the ultimate strength. The necessary condition for arising of a crack is the heating of material up to temperature above the thermal stability limit, but below the temperature of melting, when stresses decrease sharply [21].

Within the framework of the second approach to solutions to the heat problems of friction, we have analysed a distribution of transient temperature and thermal stresses in the pad (the strip) heated by a frictional heat flux whose intensity is equal to the specific power of friction in the process of braking with a constant deceleration. The solution to a corresponding problem for homogeneous semi-space has been obtained in article [22].

2 Problem formulation

Let us consider a semi-infinite foundation (the calliper) with a surface covered by a strip (the pad) of thickness d (Fig. 1). It is assumed that:

Fig. 1 Scheme of the problem

- (1) the outer surface of the strip is heated by a flux of intensity proportional to the specific friction power [13]

$$q(t) = \begin{cases} \gamma f p_0 V(t), & 0 \leq t \leq t_s, \\ 0, & t > t_s, \end{cases} \quad (1)$$

where the coefficient of friction f and the contact pressure p_0 are constant;

- (2) the heat partitioning factor γ is given by formula [2]:

$$\gamma = \frac{A_{ov} K_p \sqrt{k_d}}{A_{ov} K_d \sqrt{k_p} + K_p \sqrt{k_d}}; \quad (2)$$

- (3) the braking with a constant deceleration is under consideration when the velocity $V(t)$ decreases linearly during time:

$$V(t) = V_0 \left(1 - \frac{t}{t_s} \right), \quad 0 \leq t \leq t_s; \quad (3)$$

- (4) the perfect heat contact between the strip and the foundation takes place;
 (5) the material properties of the strip and the foundation are isotropic and independent of the temperature.

From assumption (1) follows that all mechanical energy of friction turns in thermal, i.e. we neglect a part of the mechanical energy going on wear of working surfaces of a pad and a disc. The heat partitioning factor γ in the form (2) differs from well-known Charron formula [23] presence of overlap coefficient A_{ov} . It is known that this formula can be used only when $A_{ov} \approx 1$. In the numerical analysis, the geometrical sizes of a pad and a disc are those that $A_{ov} \ll 1$. The assumption (2) that speed decreases linearly during braking is consequence of that pressure at braking reaches the nominal value p_0 very quickly (instantly). The assumption (3) is connected with absence of thermal resistance on a surface of contact of a pad and a calliper. The assumption (4) about independence of the material properties of pad and the calliper from temperature allows us to consider a corresponding thermal problem of friction during braking in linear statement. As a result, we shall find the solution of this problem in the closed analytical form.

On these assumptions for calculation of the dimensionless temperature T^* in strip and foundation, let us consider the following parabolic boundary-value problem of heat conduction:

$$\frac{\partial^2 T^*(\zeta, \tau)}{\partial \zeta^2} = \frac{\partial T^*(\zeta, \tau)}{\partial \tau}, \quad 0 < \zeta < 1, \quad \tau > 0, \quad (4)$$

$$\frac{\partial^2 T^*(\zeta, \tau)}{\partial \zeta^2} = \frac{1}{k^*} \frac{\partial T^*(\zeta, \tau)}{\partial \tau}, \quad 1 < \zeta < \infty, \quad \tau > 0, \quad (5)$$

$$\left. \frac{\partial T^*}{\partial \zeta} \right|_{\zeta=0+} = -q^*(\tau), \quad \tau > 0, \quad (6)$$

$$T^*(1+, \tau) = T^*(1-, \tau), \quad \tau > 0, \quad (7)$$

$$\left. \frac{\partial T^*}{\partial \zeta} \right|_{\zeta=1+} = K^* \left. \frac{\partial T^*}{\partial \zeta} \right|_{\zeta=1-}, \quad \tau > 0, \quad (8)$$

$$T^*(\zeta, \tau) \rightarrow 0 \quad \text{at} \quad \zeta \rightarrow \infty, \quad \tau > 0, \quad (9)$$

$$T^*(\zeta, 0) = 0, \quad 0 \leq \zeta < \infty, \quad (10)$$

where, taking the Eqs. (1–3) into account, we have

$$q^*(\tau) = \left(1 - \frac{\tau}{\tau_s}\right) H(\tau_s - \tau), \quad \tau \geq 0, \quad (11)$$

$$\zeta = \frac{z}{d}, \quad \tau = \frac{k_p t}{d^2}, \quad \tau_s = \frac{k_p t_s}{d^2}, \quad K^* = \frac{K_c}{K_p}, \quad k^* = \frac{k_c}{k_p}, \quad T^* = \frac{T - T_a}{T_0}, \quad T_0 = \frac{q_0 d}{K_p}. \quad (12)$$

3 Temperature

For solution to the boundary-value problem of heat conduction (4–10), the Laplace integral transform with respect to dimensionless time τ is applied [24]:

$$\bar{T}^*(\zeta, p) = \int_0^\infty T^*(\zeta, \tau) \exp(-p\tau) d\tau, \quad 0 \leq \zeta < \infty. \quad (13)$$

As a result, we obtain the transform of dimensionless temperature in the form:

$$\bar{T}^*(\zeta, p) = \frac{\bar{q}^*(p)}{\sqrt{p}\Delta(p)} \begin{cases} [\cosh(1 - \zeta)\sqrt{p} - \varepsilon \sinh(1 - \zeta)\sqrt{p}], & 0 \leq \zeta \leq 1, \\ \exp\left[(1 - \zeta)\sqrt{\frac{p}{k^*}}\right], & 1 \leq \zeta < \infty, \end{cases} \quad (14)$$

where

$$\Delta(p) = \frac{1}{2}(1 + \varepsilon) \exp(\sqrt{p})[1 - \lambda \exp(-\sqrt{p})], \quad (15)$$

$$\lambda = \frac{1 - \varepsilon}{1 + \varepsilon}, \quad \varepsilon = \frac{K^*}{\sqrt{k^*}}, \quad (16)$$

$\bar{q}^*(p)$ is the Laplace transform of heat flux intensity $q^*(\tau)$ (11).

The denominator $\Delta(p)$ (15) is represented in the form of geometric series

$$\frac{1}{\Delta(p)} = \frac{2}{(1 + \varepsilon)} \exp(-\sqrt{p}) \sum_{n=0}^{\infty} \Lambda^n \exp(-2n\sqrt{p}), \quad (17)$$

where

$$\Lambda^n = \begin{cases} (-1)^n |\lambda|^n, & -1 < \lambda \leq 0, \\ \lambda^n, & 0 \leq \lambda < 1, \quad n = 0, 1, 2, \dots, \end{cases} \quad (18)$$

Introducing the expansion (17), (18) into the solution (14), we obtain

$$\bar{T}^*(\zeta, p) = \frac{\bar{q}^*(p)}{\sqrt{p}} \begin{cases} \sum_{n=0}^{\infty} \Lambda^n \exp[-(2n + \zeta)\sqrt{p}] + \sum_{n=1}^{\infty} \exp[-(2n - \zeta)\sqrt{p}], & 0 \leq \zeta \leq 1, \\ \frac{2}{(1 + \varepsilon)} \sum_{n=0}^{\infty} \Lambda^n \exp\left[-(2n + 1)\sqrt{p} + (1 - \zeta)\sqrt{\frac{p}{k^*}}\right], & 1 \leq \zeta < \infty. \end{cases} \quad (19)$$

Taking into account the form of function q^* (11) and exponents under signs of sums into equations (19) and using the convolution theorem for the Laplace's transform, we consider integrals

$$J_k(\tau) = \int_0^\tau q_k^*(s) q_2^*(\tau - s) ds, \quad k = 0, 1, \quad (20)$$

where

$$q_k^*(\tau) = \left(\frac{\tau}{\tau_s}\right)^k, \quad \tau \geq 0, \quad k = 0, 1, \quad (21)$$

$$q_2^*(\tau) \equiv L^{-1}\left[\frac{1}{\sqrt{p}} \exp(-\sqrt{ap}); \tau\right] = \frac{1}{\sqrt{\pi\tau}} \exp\left[-\frac{a}{4\tau}\right], \quad a \geq 0. \quad (22)$$

Substituting functions $q_k^*(\tau)$, $k = 0, 1, 2$ (21) and (22) into the right side of Eq. (20), we find

$$J_k(\tau) = \frac{1}{\sqrt{\pi}} \int_0^\tau \left(\frac{s}{\tau_s}\right)^k \frac{1}{\sqrt{\tau-s}} \exp\left[-\frac{a}{4(\tau-s)}\right] ds, \quad k = 0, 1. \quad (23)$$

For calculation of integrals in the right side of equations (23), we use the substitution

$$\frac{a}{4(\tau-s)} = x, \quad \tau-s = \frac{a}{4x}, \quad ds = \frac{a}{4x^2} dx, \quad x \equiv X^2 = \frac{a}{4\tau} \quad \text{at } s=0, \quad x \rightarrow \infty \quad \text{at } s \rightarrow \tau. \quad (24)$$

Consequently,

$$J_0(\tau) = \frac{1}{2} \sqrt{\frac{a}{\pi}} I_0(X), \quad J_1(\tau) = \frac{1}{2\tau_s} \sqrt{\frac{a}{\pi}} \left[\tau I_0(X) - \frac{a}{4} I_1(X) \right], \quad (25)$$

where

$$I_k(X) = \int_{X^2}^\infty \frac{\exp(-x)}{x^{k+3/2}} dx, \quad k = 0, 1. \quad (26)$$

Integrating by parts, functions $I_k(X)$ are written (26) as

$$I_0(X) = \frac{2\sqrt{\pi}}{X} \text{ierfc}(X), \quad I_1(X) = \frac{2}{3} \left[X^{-3} \exp(-X^2) - I_0(X) \right], \quad (27)$$

where

$$\text{ierfc}(X) = \frac{1}{\sqrt{\pi}} \exp(-X^2) - X \text{erfc}(X), \quad \text{erfc}(X) = 1 - \text{erf}(X), \quad (28)$$

$\text{erf}(X)$ is the error function. Substituting functions $I_k(X)$, $k = 0, 1$ (28) into formulae (25), we obtain finally

$$J_0(\tau) = 2\sqrt{\tau} \text{ierfc}(X), \quad J_1(\tau) = 2\sqrt{\tau} \left(\frac{\tau}{\tau_s}\right) F(X), \quad (29)$$

where

$$F(X) = \frac{1}{3} \left[2(1 + X^2) \text{ierfc}(X) - X \text{erfc}(X) \right]. \quad (30)$$

Taking the functions $J_k(X)$, $k = 0, 1$ (29, 30) into account, from transform solutions (19), we find the dimensionless temperatures $T^{(k)*}(\zeta, \tau)$, $k = 0, 1$ for the intensities of heat fluxes $q_k^*(\tau)$, $k = 0, 1$ (21) separately

$$T^{(k)*}(\zeta, \tau) = \sum_{n=0}^{\infty} \Lambda^n T_n^{(k)*}(\zeta, \tau), \quad \tau \geq 0, \quad k = 0, 1, \quad (31)$$

where

$$T_n^{(0)}(\zeta, \tau) = \begin{cases} u \left[\text{ierfc}\left(\frac{2n+\zeta}{u}\right) + \text{ierfc}\left(\frac{2n-\zeta}{u}\right) \right], & 0 \leq \zeta \leq 1, \\ \frac{2u}{(1+\varepsilon)} \left\{ \text{ierfc}\left[\frac{(2n+1)}{u} + \frac{(\zeta-1)}{u\sqrt{k^*}}\right] \right\}, & 1 \leq \zeta < \infty, \end{cases} \quad (32)$$

$$T_n^{(1)}(\zeta, \tau) = \begin{cases} u \left(\frac{u}{u_s}\right)^2 \left[F\left(\frac{2n+\zeta}{u}\right) + F\left(\frac{2n-\zeta}{u}\right) \right], & 0 \leq \zeta \leq 1, \\ \frac{2u}{(1+\varepsilon)} \left(\frac{u}{u_s}\right)^2 F\left[\frac{(2n+1)}{u} + \frac{(\zeta-1)}{u\sqrt{k^*}}\right], & 1 \leq \zeta < \infty, \end{cases} \quad (33)$$

$$u = 2\sqrt{\tau}, \quad u_s = 2\sqrt{\tau_s} \quad (34)$$

Knowing the temperature fields $T^{(k)*}(\zeta, \tau)$, $k = 0, 1$ (31–34), the dimensionless temperature $T^*(\zeta, \tau)$ of the strip and the foundation in the arbitrary moment of dimensionless time $\tau \geq 0$ (from initial heating in braking start to total cooling after stop) may be presented as the superposition [11, 12]

$$T^*(\zeta, \tau) = T^{(0)*}(\zeta, \tau) - T^{(1)*}(\zeta, \tau) + T^{(1)*}(\zeta, \tau - \tau_s)H(\tau - \tau_s), \quad 0 \leq \zeta < \infty, \quad \tau \geq 0. \quad (35)$$

In the limit case $\tau_s \rightarrow \infty$ at fixed τ from Eq. (33), it follows that $T_n^{(1)} \rightarrow 0$ ($T^{(1)*} \rightarrow 0$), and from Eqs. (31, 32, 34 and 35), we obtain the solution of the transient problem of heat conduction for the strip-foundation system in the case of heating the free surface of the strip by a heat flux of constant intensity [18].

4 Thermal stresses

It is known that the normal component of stresses σ_z is insignificant in the case of plane problem. The quasi-static longitudinal σ_x and transverse σ_y normal stresses in the strip initiated by transient temperature field $T^*(\zeta, \tau)$ (35) are possible to determine by formulas of temperature bending of thick plate with unfixed ends [25]:

$$\sigma_x(z, t) = \sigma_y(z, t) = \sigma_0 \sigma^*(\zeta, \tau), \quad 0 \leq z \leq d, \quad t \geq 0 \quad (36)$$

where

$$\sigma^*(\zeta, \tau) = \varepsilon^*(\zeta, \tau) - T^*(\zeta, \tau), \quad 0 \leq \zeta \leq 1, \quad \tau \geq 0 \quad (37)$$

$$\varepsilon^*(\zeta, \tau) = \int_0^1 T^*(\zeta, \tau) d\zeta + 12(\zeta - 0.5) \int_0^1 (\zeta - 0.5) T^*(\zeta, \tau) d\zeta, \quad 0 \leq \zeta \leq 1, \quad \tau \geq 0 \quad (38)$$

Taking the solution of temperature (31–35) into account, the dimensionless temperature strain ε^* (38) can be written in form:

$$\varepsilon^*(\zeta, \tau) = \varepsilon^{(0)*}(\zeta, \tau) - \varepsilon^{(1)*}(\zeta, \tau) + \varepsilon^{(1)*}(\zeta, \tau - \tau_s)H(\tau - \tau_s), \quad 0 \leq \zeta \leq 1, \quad \tau \geq 0 \quad (39)$$

where

$$\varepsilon^{(k)*}(\zeta, \tau) = \sum_{n=0}^{\infty} \Lambda^n \varepsilon_n^{(k)*}(\zeta, \tau), \quad 0 \leq \zeta \leq 1, \quad \tau \geq 0, \quad k = 0, 1 \quad (40)$$

$$\varepsilon_n^{(k)*}(\zeta, \tau) = Q_n^{(k)}(\tau) - \zeta R_n^{(k)}(\tau), \quad k = 0, 1 \quad (41)$$

$$Q_n^{(k)}(\tau) = 4I_n^{(k)}(\tau) - 6J_n^{(k)}(\tau), \quad R_n^{(k)}(\tau) = 6I_n^{(k)}(\tau) - 12J_n^{(k)}(\tau), \quad k = 0, 1; \quad n = 0, 1, 2, \dots, \quad (42)$$

$$I_n^{(k)}(\tau) = \int_0^1 T_n^{(k)*}(\zeta, \tau) d\zeta, \quad J_n^{(k)}(\tau) = \int_0^1 \zeta T_n^{(k)*}(\zeta, \tau) d\zeta, \quad k = 0, 1; \quad n = 0, 1, 2, \dots \quad (43)$$

Substituting the functions $T_n^{(k)*}(\zeta, \tau)$ (32–34) into the relations (43), we obtain

$$I_0^{(k)}(\tau) = C^{(k)}(u) L^{(k)}\left(\frac{1}{u}\right), \quad J_0^{(k)}(\tau) = u C^{(k)}(u) M^{(k)}\left(\frac{1}{u}\right), \quad k = 0, 1 \quad (44)$$

$$I_n^{(k)}(\tau) = C^{(k)}(u) \left[L^{(k)}\left(\frac{2n+1}{u}\right) - L^{(k)}\left(\frac{2n-1}{u}\right) \right], \quad k = 0, 1; \quad n = 1, 2, 3, \dots, \quad (45)$$

$$J_n^{(k)}(\tau) = C^{(k)}(u) \left\{ u \left[M^{(k)}\left(\frac{2n+1}{u}\right) - 2M^{(k)}\left(\frac{2n}{u}\right) + M^{(k)}\left(\frac{2n-1}{u}\right) \right] - 2n \left[L^{(k)}\left(\frac{2n+1}{u}\right) - 2L^{(k)}\left(\frac{2n}{u}\right) + L^{(k)}\left(\frac{2n-1}{u}\right) \right] \right\}, \quad k = 0, 1; \quad n = 1, 2, 3, \dots \quad (46)$$

$$C^{(0)}(u) = u^2, \quad C^{(1)}(u) = 0.25u^4 \quad (47)$$

where [26,27]

$$L^{(0)}(x) \equiv \int_0^x \text{ierfc}(s) ds = \frac{1}{4} + \frac{x}{2\sqrt{\pi}} \exp(-x^2) - \frac{(1+2x^2)}{4} \text{erfc}(x), \quad (48)$$

$$M^{(0)}(x) \equiv \int_0^x s \text{ierfc}(s) ds = \frac{1}{6\sqrt{\pi}} - \frac{(1-2x^2)}{6\sqrt{\pi}} \exp(-x^2) - \frac{x^3}{3} \text{erfc}(x), \quad (49)$$

$$L^{(1)}(x) \equiv \int_0^x F(s) ds = \frac{1}{8} + \frac{(5+2x^2)}{12\sqrt{\pi}} \exp(-x^2) - \frac{(3+12x^2+4x^4)}{24} \text{erfc}(x), \quad (50)$$

$$M^{(1)}(x) \equiv \int_0^x s F(s) ds = \frac{1}{15\sqrt{\pi}} - \frac{(1-4x^2-2x^4)}{15\sqrt{\pi}} \exp(-x^2) - \frac{x^3(5+2x^2)}{15} \text{erfc}(x). \quad (51)$$

5 The homogeneous semi-space

To verify the correctness of the obtained solutions, we shall try to receive the known solutions to a homogeneous semi-space. In the case of identical physical properties of a strip and a foundation ($K_p = K_c$, $k_p = k_c$), from formulae (12, 16 and 18), it follows that $K^* = 1$, $k^* = 1$, $\varepsilon = 1$, $\lambda = 0$, $\Lambda = 0$, and from equations (31–34) at $n = 0$, we obtain the solution to the problem of frictional heating during braking with constant deceleration for homogeneous semi-space [22]:

$$T^{(k)*}(\zeta, \tau) = 2\sqrt{\tau} \left(\frac{\tau}{\tau_s} \right)^k F^{(k)} \left(\frac{\zeta}{2\sqrt{\tau}} \right), \quad 0 \leq \zeta < \infty, \quad \tau \geq 0, \quad k = 0, 1 \quad (52)$$

Substituting the solution (52) at $\zeta = 0$, $0 \leq \tau \leq \tau_s$ into the right side of equation (35), we obtain the Fazekas known formula for the calculation of dimensionless contact temperature [4]:

$$T^*(0, \tau) = 2\sqrt{\frac{k^*\tau}{\pi}} \left(1 - \frac{2\tau}{3\tau_s} \right), \quad 0 \leq \tau \leq \tau_s. \quad (53)$$

Taking the function $I_0^{(k)}(\tau)$, $J_0^{(k)}(\tau)$, $k = 0, 1$ (44) into account, from formulae (42), it follows that

$$Q_0^{(0)}(\tau) = u^2 \left[1 - \frac{u}{\sqrt{\pi}} + u \text{ierfc} \left(\frac{1}{u} \right) \right], \quad (54)$$

$$R_0^{(0)}(\tau) = u^2 \left[\frac{3}{2} - \frac{2u}{\sqrt{\pi}} + \frac{u}{2\sqrt{\pi}} \exp \left(-\frac{1}{u^2} \right) + \left(\frac{3u}{2} - \frac{1}{u} \right) \text{ierfc} \left(\frac{1}{u} \right) \right], \quad (55)$$

$$Q_0^{(1)}(\tau) = \frac{u^4}{4} \left[\frac{1}{2} - \frac{2u}{5\sqrt{\pi}} - \frac{1}{5\sqrt{\pi}} \left(\frac{u}{2} - \frac{1}{3u} \right) \exp \left(-\frac{1}{u^2} \right) + \left(\frac{u}{2} - \frac{2}{15u^3} \right) \text{ierfc} \left(\frac{1}{u} \right) \right], \quad (56)$$

$$R_0^{(1)}(\tau) = \frac{u^4}{4} \left[\frac{3}{4} - \frac{4u}{5\sqrt{\pi}} + \frac{1}{10\sqrt{\pi}} \left(\frac{u}{2} + \frac{3}{u} \right) \exp \left(-\frac{1}{u^2} \right) + \left(\frac{3u}{4} - \frac{1}{u} - \frac{3}{5u^3} \right) \text{ierfc} \left(\frac{1}{u} \right) \right]. \quad (57)$$

The Eqs. (37–41) at $n = 0$ with functions $Q_0^{(k)}(\tau)$, $R_0^{(k)}(\tau)$, $k = 0, 1$ (54–57) allow to determine the non-stationary dimensionless stresses $\sigma^*(\zeta, \tau)$ in a homogeneous semi-space [22].

6 Numerical analysis

The numerical results have been computed for the commercial frictional material FMK-11 of the pad (the strip) on the steel calliper (the foundation), for which [2]:

$$\text{FMK-11: } K_p = 34.3 \text{ W m}^{-1} \text{ K}^{-1}, \quad k_p = 15.2 \cdot 10^{-6} \text{ m}^2 \text{ s}^{-1},$$

Fig. 2 Isotherms of temperature $T(z, t)$

30KhGSA steel calliper: $K_c = 37.2 \text{ Wm}^{-1} \text{ K}^{-1}$, $k_c = 10.3 \cdot 10^{-6} \text{ m}^2 \text{ s}^{-1}$.

The ceramic-metal frictional material FMK-11 consists of 64% Fe, 15% Cu, 9% C, 3% SiO_2 , 3% asbestos and 6% BaSO_4 [2]. The metal components of FMK-11 (Fe, Cu) provide to a material high heat conductivity and wear-in, and the non-metallic component (C, SiO_2 , et al.) increase the coefficient of friction and reduce propensity to jamming. The frictional material on the basis of FMK-11 is intended for work in the hard loaded wheel disc brakes of planes [6].

The chemical composition of steel 30KhGSA is as follows (in %): $0.9 \div 1.2$ Si, $0.8 \div 1.1$ Mn and Cr, not more than 0.3 Cu and Ni, not more than 0.025 P and S [28]. Constructional steel 30 KhGSA is applied in various areas of mechanical engineering, for example, in aircraft construction for creation of details that are supposed to be used there where high loading and adverse conditions is possible: these are the fixing details working at low temperatures, the welded designs testing sign-variable loadings, etc.

The thermo-physical properties of the friction materials and the operating friction conditions $p_0 = 1 \text{ MPa}$, $f = 0.7$, $V_0 = 30 \text{ ms}^{-1}$, $t_s = 3.44 \text{ s}$, $d = 5 \text{ mm}$, $T_a = 20^\circ \text{C}$ are the same as used to numerical analysis in solution to the contact problem for three-element tribosystem (disc/pad/calliper) in article [11]. The geometrical sizes of friction surface of the pad and the disc are equal $A_{a1} = 0.005 \text{ m}^2$, $A_{a2} = 0.0264 \text{ m}^2$ [2].

The calculations are carried out according to the following scheme:

1. by formulae (12), we calculated the dimensionless input parameters of the problem: ζ , τ , τ_s , K^* , k^* ;
2. the dimensionless transient temperature T^* in the strip and foundation is defined from solutions (31–35).
3. the dimensionless thermal stresses σ^* can be found from solution (37–51),
4. on the assumption that the pad slides on the surface of the cast-iron disc ($K_d = 51 \text{ Wm}^{-1} \text{ K}^{-1}$, $k_d = 14 \cdot 10^{-6} \text{ m}^2 \text{ s}^{-1}$), we obtain the heat participation factor $\gamma = 0.147$ (2), which means that during braking 85.3% of heat flux intensity is directed to the disc, and 14.7% - to the pad;
5. by formulae (12), we calculated the dimensional values: the intensity of heat flux $q_0 = \gamma f p_0 V_0$, the parameter T_0 and finally, the temperature $T = T_a + T_0 T^*$.

We checked the process of computing the accuracy of summation in series (31 and 40) by comparing results for the two values of parameter of summation: n and $2n$. The calculations are stopped if the absolute value of the difference calculation is less than the input tolerance.

Isolines for the temperature constructed in the coordinate system (z, t) are shown in Fig. 2. The maximal temperatures $T_{\max} = 751^\circ \text{C}$ are reached on a contact surface $z = 0$ at the time moments $t_{\max} = 1.6 \text{ s}$.

Fig. 3 Evolution of the contact temperature $T(0, t)$ at $t_s = 3.44s$

Fig. 4 Dependence of the maximal temperature $T_{\max} = T(0, t_{\max})$ on the strip thickness d

This result corresponds with the experimental data $T_{\max} = 760^{\circ}\text{C}$ published in monograph [6]. The obtained value of maximal temperature on a surface of heating is in good agreement with the corresponding numerical results $T_{\max} = 740^{\circ}\text{C}$ of the article [11], when the solution to a heat problem of friction for disc/pad/calliper tribosystem is obtained in a contact statement.

The evolution of the temperature on the friction surface $z = 0$ is shown in Fig. 3. With the beginning of braking, the temperature sharply increases, reaching the above mentioned maximal value $T_{\max}$ in the moment of time $t_{\max}$. Further, up to the end of braking, the temperature decreases. After a stop, cooling of a surface of friction proceeds.

Dependences of the maximal temperature $T_{\max}$ on the strip thickness d are shown in Fig. 4. The influence of the thickness on the maximal temperature on the contact surface is significant to $d \leq 10$ mm for a FMK-11 pad (Fig. 4). At thickness of a pad greater than these values, the maximal temperature may be calculated using the Fazekas solution (53) for homogeneous semi-space.

The maximal value of the temperature $T_{\max}$ rises nonlinearly when the braking time increases. This dependence can be described approximately as

Fig. 5 Dependence of the maximal temperature $T_{\max} = T(0, t_{\max})$ on the time of braking t_s

Fig. 6 Isolines of the dimensionless normal stress $\sigma^*(z, t)$ at $t_s = 3.44s$

$$T_{\max} = 4.026t_s^3 - 54.121t_s^2 + 358.49t_s - 5.971 \text{ (Fig. 5).}$$

Isolines of the dimensionless normal stresses σ^* (37) in the strip are presented in Fig. 6. It is observed that in the time interval $0 \leq t \leq 2.2s$, the region of compressive stress occur near the surface of friction $z = 0$. Inside the strip, the region of tensile stresses arises. Between the region with compressive and tensile stresses, there are two isolines with zero stresses. The third line of zero stresses, which “descends” from the surface $z = 0$, appears when the heating time is equal $t_c \approx 2.4s$, and it is shorter than time of braking $t_s = 3.44s$. It means that during relaxation phase at $t > t_s$, when there is no more heating, the sign of stresses does not change and the region of tensile stresses expands further from the heated surface—the line of zero stresses moves parallel to the surface of friction with the increase of time.

Dependence of the time t_c , when the normal stress on a surface of friction becomes tensile, from thickness d of the strip is shown in Fig. 7. With an increase in thickness of a strip, the time of initiation of tensile stresses on its surface increases.

The initiation of superficial cracks generation is accompanied with the monotonic increase in tensile normal stresses. In heating of frictional elements during braking, it is the change of sign of superficial stresses that plays key role in forecasting initiation and preventive maintenance of thermal splitting [29]. Change in time of the dimensionless normal stress σ^* (37) on the surface of heating is shown in Fig. 8. We see that the tensile stresses reach the maximum value at the time of the stop at $t = t_s = 3.44s$. After stopping in the period of cooling, tensile stresses decrease with time.

Fig. 7 The time t_c versus thickness d of the strip at $t_s = 3.44$ s

Fig. 8 Evolution of the dimensionless normal stress $\sigma^*(0, t)$ on the contact surface

7 Conclusions

The approximate analytical method for determination of temperature and thermal stresses in a pad during braking is proposed. We have made two main simplifying assumptions of this method. The first is the assumption of one-dimensional process of heat conduction, and the second is an introduction to the consideration of partition ratio of frictional thermal fluxes. This second assumption allows to solve a heat problem of friction during braking for a pad and a disc separately. We have investigated a transient temperature field and thermal stresses in the pad only.

The numerical analysis is executed for FMK-11 material of a pad. For these materials, it is established that:

- the temperature of the FMK-11 material of the pad increases significantly within its thickness during braking;
- for the values of the pad thickness $d > 10$ mm for FMK-11 material, the maximal temperature on the friction surface may be calculated using the Fazekas formula (53);
- the time of initiation of tensile stresses on the surface of friction increases with an increase of thickness of a pad.

The obtained value of maximal temperature on a surface of heating is in good agreement with the corresponding numerical results of the article [11] and with the experimental data of the monograph [6]. We have also confirmed the conclusions of paper [12] that the tensile stresses are initiated near to a surface of frictional elements during braking.

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