Arithmetic

by Paul Lockhart

REVIEWED BY PAMELA GORKIN

A musician wakes from a terrible nightmare. In his dream he finds himself in a society where music education has been made mandatory. "We are helping our students become more competitive in an increasingly sound-filled world." Educators, school systems, and the state are put in charge of this vital project. Studies are commissioned, committees are formed, and decisions are made—all without the advice or participation of a single working musician or composer ...

Meanwhile, on the other side of town, a painter has just awakened from a similar nightmare ... I was surprised to find myself in a regular school classroom—no easels, no tubes of paint. "Oh we don't actually apply paint until high school," I was told by the students ...

Sadly, our present system of mathematics education is precisely this kind of nightmare.

So begins Paul Lockhart's *A Mathematician's Lament*.[1] I was charmed by that essay and was looking forward to his newest book, *Arithmetic*. Though this is not a book about education, it has an innovative take on arithmetic. Perhaps, as Lockhardt hopes, the reader will not read *Arithmetic* because it is mandatory, but rather to "experience firsthand the simple pleasure and satisfaction that comes with numerical fluency."

Starting with tally marks and groupings, Lockhart sets the stage for the early part of the book: Why do we group numbers as we do? Why do we work with our particular symbols for numbers? He imagines three tribes: the Hand People, who communicate with hand gestures (four claps for four, one thump for five); the Banana People, who express their numbers verbally and group them in fours (*na* is one, *na na* is two, *ba* is four); and the Tree People, who communicate using scratches on bark. How do these tribes communicate with each other? From this, we pass to Egypt, and Lockhart asks us to think about what would happen if the Banana People were conquered by the Egyptians. Then we learn about Rome. Here Lockhart explains that the Romans had a kind of abacus, called a *tabula*. He suggests that the reader make one and then poses the following problem: How would you design a tabula for the Banana People? From there, it's on to China and Japan, India, and Europe, and then we are ready for multiplication, division, and fractions. But what are the advantages of using a particular system? And what are the disadvantages? *Arithmetic* tells us that too. The book ends with a look at negative numbers and a brief final chapter on counting, which includes such familiar territory as counting the number of possible license plates in California.

These discussions include a collection of exercises for the reader, from the elementary, such as "Make a list of things that you think of as smooth or lumpy. What are some things that can be treated both ways?" to problems that are, for the reader at the right level, really interesting: "Define the midpoint operator $\diamond$ to be the number halfway between two numbers (e.g., $3 \diamond 5 = 4$, $9 \diamond 3 = 6$, $4 \diamond 1 = 5/2$). Is this operation associative?"

In addition to the exercises, another attractive feature of the book is the illustrations. For example, the difference between 3×8 (looking at three groups of eight) and 8×3 (eight groups of three) is illustrated with what appear to be small hand-drawn pictures. Though references would have been appreciated and the reader will have to take the author's word for many historical statements, the book is written clearly, has accessible explanations, and is beautifully packaged. This is an ideal book for someone looking for new ways to interest a young student in mathematics.

Department of Mathematics
Bucknell University
Lewisburg, PA 17837
USA
e-mail: pgorkin@bucknell.edu

---

[1]*A Mathematician's Lament: How School Cheats Us Out of Our Most Fascinating and Imaginative Art Form.* Bellevue Literary Press, 2009.