Bridge to Abstract Mathematics

by Ralph W. Oberste-Vorth, Aristides Mouzakitis, and Bonita A. Lawrence

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The transition from calculation-oriented school mathematics to writing proofs for theorems in abstract mathematics is not easy for anyone. The Calculus Reform in the United States and similar changes in the curricula in other countries have only postponed the difficulty that undergraduate students encounter in crossing the bridge. No wonder many academic institutions have inserted special courses into their curricula to address this problem. *Bridge to Abstract Mathematics*, by Ralph W. Oberste-Vorth, Aristides Mouzakitis, and Bonita A. Lawrence, is intended as a primary text for such a course.

There exist a number of older textbooks written with a similar intention, for instance Ronald P. Morash's *Bridge to Abstract Mathematics: Mathematical Proof and Structures* (1991) and Daniel J. Velleman's *How to Prove It: A Structured Approach* (1994). For natural reasons, such books often have overlapping contents and are even organized in a comparable manner, so one may ask whether the present book adds anything new to the topic.

Having given a one-semester course in mathematics teachers' education on proofs and fundamentals of advanced mathematical thinking using this book, my answer is positively "yes." First, in my experience, the modest scope of a primary textbook makes a difference in how receptive students are in such a first course. *Bridge to Abstract Mathematics* has 232 pages, and it focuses exclusively on building a bridge to abstract mathematics; it refrains from causing stress to unseasoned students by overly advertising between the lines all that is yet to be learnt beyond that bridge.

Second, and more important, in addition to properly discussing the fundamentals of sets, elementary logic, the idea of a proof, relations, functions, and the most common number systems, some place has been made for "Absurda and poofs," i.e., false propositions and failed proofs. According to modern learning theories, e.g., [1], variation is a crucial prerequisite for learning. In order to thoroughly understand, say, the concept of a monotone function, a learner must have a varied experience of both monotone functions and those that are neither increasing nor decreasing [2]. Similarly, a full

comprehension of what constitutes a valid proof also requires paying attention to justifications that do not work. Further, attention is paid to the existence of alternative proofs, which reveals another dimension of variation often neglected in school mathematics: most mathematical facts can be established in several ways.

I also enjoyed reading the more informal commentary sections where the authors explain the background of mathematical notions, reflect on their own and the students' learning processes, and on the mathematical parlance in general. Interestingly, a rather modest use of illustrations is made throughout the book. For instance, there are no figures in the chapters discussing limit, continuity, and derivative.

What I might have done differently is the construction of the natural numbers. The given definition that is based on the successors and immediate predecessors is, of course, correct as such, but since the authors dwell for quite many pages on sets and equivalence relations, another approach built on cardinality of sets—where, for example, the addition of two natural numbers is established through the union of two disjoint sets—might have resulted in a more coherent whole.

The chosen examples and exercises are quite ordinary in their focusing on proving the standard facts. Solving a few linear equations or doing some sums in detail first in the context of the natural numbers, and then discussing the same problems in the context of the integers and the rationals could have increased the value of this book. The inclusion of such items would demonstrate on a practical level how radically different these number systems ultimately are even if we most often consider them as merely being nested within each other, at least in school mathematics.

In general, *Bridge to Abstract Mathematics* is readable and even entertaining at its best. The first version of the text seems to date from the turn of the millennium, and several drafts have been tested since. As a result, only a few flaws have remained in the printed version. I believe that I shall use this book also in the next year's course.

REFERENCES

- [1] F. Marton and S. Booth, *Learning and Awareness*. Lawrence Erlbaum Associates, Mahwah, NJ, 1997.
- [2] T. Tossavainen, P. Haukkanen, and M. Pesonen, Different aspects of the monotonicity of a function. *International Journal of Mathematical Education in Science and Technology* (2013), DOI: [10.1080/0020739X.2013.770088](https://doi.org/10.1080/0020739X.2013.770088).

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