Three Thoughts on "Prime Simplicity"

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The Mathematical Intelligencer *encourages comments about the material in this issue.* Letters *should be sent to either of the editors-in-chief, Chandler Davis or Marjorie Senechal.*

In 2009, Catherine Woodgold and I published "Prime Simplicity" [2], examining the belief that Euclid's famous proof of the infinitude of prime numbers was by contradiction. We demonstrated that that belief is widespread among mathematicians and is false: Euclid's proof is simpler and better than the frequently seen proof by contradiction. The extra complication of the indirect proof serves no purpose and has pitfalls that can mislead the reader.

Dirichlet

The many examples we cited were all from sources since 1900. This cutoff date was not planned. We set out to document modern views. If we had set out to trace the history of the misunderstanding, we might not have missed a gem pointed out by Robert J. Gray: like many later authors, J. P. G. Lejeune-Dirichlet, in a posthumous book [4, pages 9–10], falsely attributed the proof by contradiction to Euclid. Could all those twentieth-century occurrences of the error stem from Dirichlet? That question I leave open.

Square Roots and Contradictions

We noted that neither we nor Euclid objected to proofs by contradiction in general, and in particular Euclid proved the irrationality of $\sqrt{2}$ by contradiction. Later, on page 46, we said that that fact is "a negative result that can only be proved by contradiction." Not so, say Karin Usadi Katz and Mikhail Katz! They write [3, pages 13–14],

> Without exploiting the hypothetical equality $\sqrt{2} = \frac{m}{n}$, one can exhibit positive lower bounds for the difference $|\sqrt{2} - \frac{m}{n}|$ in terms of the denominator n, resulting in a constructively adequate proof of irrationality.

In a footnote, they give the lower bound $1/(3n^2)$.

Fortunately, our statement about $\sqrt{2}$ was in no way essential to our theses.

Chronology

Finally, I would like to clarify something that might be confusing. The paper as submitted to this journal contrasted a passage [1, pages 122–123] written by G. H. Hardy more than a hundred years ago with "Euclid's proof as presented by Øystein Ore above." The word "above" meant earlier in our paper, where Ore's paraphrase [5, page 65] of the proof was quoted in its entirety. Some copyeditor changed "above" to "earlier." I objected to the change on the grounds that it makes it appear that we were saying Ore's 1948 book appeared earlier than G. H. Hardy's 1908 book. I was told that "above" would appear in the published paper. It didn't. For the record, my coauthor and I were aware that 1948 is not earlier than 1908.

REFERENCES

[1] Hardy, G. H., *A Course of Pure Mathematics*, Cambridge University Press, 1908.

[2] Hardy, M. and Woodgold, C., ''Prime Simplicity,'' *Mathematical Intelligencer* 31 (2009), no. 4, 44–52.

[3] Katz, K. U. and Katz, M., ''Meaning in Classical Mathematics: Is It at Odds with Intuitionism?'' <http://arxiv.org/pdf/1110.5456v1.pdf>.

[4] Lejeune-Dirichlet, J. P. G., *Lectures on Number Theory*, American Mathematical Society, 1999 (translation by John Stillwell of *Vorlesungen über Zahlentheorie*, Friedrich Vieweg und Sohn, 1863).

[5] Ore, Ø., Number Theory and Its History, Courier Dover Publications, 1988 (reprint of a book published by McGraw–Hill in 1948).

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