The Persian Tarof in Mathematics

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I was prevailed upon by my colleague's recent brief proof of the AM-GM inequality in [1], to give some comments on this proof. In my opinion this proof should not just be considered as another simple proof of this inequality, for it has some conspicuous advantages over the other proofs of this inequality in the literature. First, it recasts one of the best understood statements regarding the positive real numbers (i.e., $a \leq x \leq b$), which is written by everyone who is dealing with mathematics in any way more than thousands of times in each one's life, for different purposes, into a useful unprecedented statement (i.e., $a + b \geq x + ab/x$). In fact, the latter trivial inequality is the whole proof of the AM-GM inequality in [1]. Second, this trivial inequality leads us to infinitely many nontrivial inequalities in two variables, in a trivial way. For example, by taking $x = \sqrt[n+m]{a^m b^n}$, where m, n are positive integers, we get $a + b \geq \sqrt[n+m]{a^m b^n} + \sqrt[n+m]{a^m b^n}$, or if in general we take $x = f(a, b)$, where f is a suitable function, then we get nontrivial inequalities in terms of a, b . Also, if we assume that $0 < a \leq x < y \leq b$, then we can easily show that

$$\frac{1}{2}(x+y) + ab \frac{\ln(y) - \ln(x)}{y-x} \leq a+b.$$

To see this, recall that $t + \frac{ab}{t} \leq a+b$, for $a \leq t \leq b$. This immediately implies that $\int_x^y (t + \frac{ab}{t}) dt \leq (y-x)(a+b)$, hence we get $\frac{1}{2}(y^2 - x^2) + ab(\ln(y) - \ln(x)) \leq (y-x)(a+b)$, and if we divide both sides of the latter inequality by $y-x$, we get the above inequality (Note, it seems that one cannot easily obtain this inequality by using the Mean Value Theorem). Third, as mentioned in the remark in [1], it

settles the equality case in the AM-GM inequality automatically. Finally, as for the title of my letter, when the other day I was discussing these comments with Professor Karamzadeh, although he quite approved the comments, when I said that I had better send these comments to *The Mathematical Intelligencer* as a letter to the Editors, he suddenly tried to dissuade me from doing this. He later told me, "Don't you ever care what the other people think when they find out a colleague of mine who also happens to be an ex-student of mine, is somehow praising my work? Or doesn't this reflect an all-too-high opinion of ourselves?" And he went on, saying that, "In contrast to the expression 'a pompous politician' we should not let the corresponding expression 'a pompous mathematician' find any place in the literature." And at the end, while he was walking away, he reminded me of a quoted saying of Albert Einstein ("Most people say that it is the intellect which makes a great scientist. They are wrong: it is character"), but I replied without delay, "Don't you think that Einstein is, in a way, doing a kind of Persian tarof in this statement?" But he just laughed it off. What he did to dissuade me is a Persian custom, which is called "tarof" in Farsi (i.e., when offered something in one's favor, you must refuse, even when you would like to accept it), but he was doing this in an excessive way (note, a person with this habit is called "tarofi" in Persian). But, I as an Iranian must admit, I did not know, as yet, that the Persian tarof applies even to mathematical results, too, and at the same time I will be glad, if my letter is published, that with the help of mathematics the meaning of the Persian tarof would become clear to many people, even to non-Persians who know a little mathematics. Therefore, with all due respect to Karamzadeh, I decided to try publishing this letter in *The Mathematical Intelligencer*, in order to, at least, let others know more about our Persian tarof and that the Persians are not the only people who do tarof. (Note, Einstein is not an Iranian, although I wish he were.) It seems, I am doing a kind of Persian tarof inside those parentheses, aren't I?

REFERENCE

- [1] O. A. S. Karamzadeh, One-line proof of the AM-GM inequality, *Mathematical Intelligencer* 33 (2) (2011), 3.

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