Erratum to: Risk-Sensitive Control with Near Monotone Cost

Anup Biswas · V.S. Borkar · K. Suresh Kumar

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We have used Theorem 3 of [2] in the proof of Theorem 2.1 in order to claim $\rho \leq \beta$, and also in a remark preceding Theorem 2.2. The proof thereof in [2], however, is flawed. Our own proof of the reverse inequality $\rho \geq \beta$ also has gaps. We give an alternative proof of $\rho = \beta$ below that sidesteps both difficulties.

Let χ_n denote a nonnegative smooth function such that $\chi_k \equiv 1$ in $B_k := \{x : \|x\| \leq k\}$, $\chi_k \equiv 0$ in B_{k+1}^c and $0 \leq \chi_k \leq 1$. Let $r_k = r\chi_k$. Define for $\alpha > 0$,

$$u_k^\alpha(\theta, x) := \inf_{v \in M_1} E_x[e^{\theta \int_0^\infty e^{-\alpha t} r_k(X_t, v_t) dt}].$$

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A. Biswas

Tata Institute of Fundamental Research, Centre for Applicable Mathematics, Bangalore 560065,
India

e-mail: anup@math.tifrbng.res.in

V.S. Borkar (✉)

School of Technology and Computer Science, Tata Institute of Fundamental Research, Mumbai
400005, India

e-mail: borkar@tifr.res.in

K. Suresh Kumar

Department of Mathematics, Indian Institute of Technology Bombay, Mumbai 400076, India

e-mail: suresh@math.iitb.ac.in

Let τ^v denote the entrance time to the ball B_{k+1} under the Markov control $v \in M_1$. Let $x \in B_{k+1}^c$. Then standard dynamic programming argument gives

$$u_k^\alpha(\theta, x) \leq \inf_{M_1} E_x[u_k^\alpha(\theta, X_{\tau^v})] \leq \sup_{y \in \partial B_{k+1}} u_k^\alpha(\theta, y).$$

Here we used the facts that $r_k \equiv 0$ on B_{k+1}^c and $e^{-\alpha\tau^v} < 1$ a.e. By closely mimicking the arguments of Theorem 2.1, one has

$$\log u_k^\alpha(\theta, x) - \log u_k^\alpha(\theta, 0) \rightarrow \phi^k(\theta, x)$$

in $W_{loc}^{1,p}(\mathbb{R}^n)$, $p \geq 2$, along a subsequence. By the Sobolev embedding theorem, the convergence is uniform on compact subsets of $\mathbb{R}^n$. But the foregoing arguments show that for $x \in B_{k+1}^c$,

$$\begin{aligned} \log u_k^\alpha(\theta, x) - \log u_k^\alpha(\theta, 0) &\leq \sup_{y \in \partial B_{k+1}} [\log u_k^\alpha(\theta, y) - \log u_k^\alpha(\theta, 0)] \\ &\Rightarrow \phi^k(\theta, x) \leq K(k), \end{aligned}$$

where $K(k)$ can be chosen independent of $x \in \mathbb{R}^n$. Set $\hat{u}^k = e^{\theta\phi^k}$. We set $\theta = 1$ without loss of generality and suppress the θ -dependence now on for notational simplicity. Using arguments as in the proof Theorem 2.1, we have $(\hat{u}^k, \rho_k)$ satisfying

$$\rho_k \hat{u}^k = \inf_{v \in V_1} [L\hat{u}^k(x, v) + r_k(x, v)\hat{u}^k], \quad (1)$$

with $\hat{u}^k(0) = 1$. Moreover, $\hat{u}^k$ is bounded above by $e^{K(k)}$. This fact allows us to show, using a standard argument based on Ito's formula, that

$$\rho_k \leq \inf_{T \rightarrow \infty} \limsup \frac{1}{T} \log E \left[e^{\int_0^T (r_k(X_t, v_t)) dt} \right], \quad (2)$$

where the infimum is over all admissible controls. Since $|r_k| \leq |r|$, ρ_k 's are uniformly bounded in k . Since $\hat{u}^k(0) = 1$ and the coefficients of (1) are bounded, we have $\hat{u}^k$ uniformly bounded on compact sets by Harnack's inequality (p. 199, [1]). Thus using Theorem 9.11, page 235, [1], we have $\hat{u}_k \rightarrow \hat{u}$ in $W_{loc}^{1,p}(\mathbb{R}^n)$ and $\rho_k \rightarrow \rho$ along a subsequence. Furthermore, it follows from Harnack's inequality that $\hat{u} > 0$, in fact one has uniform positive lower bounds on $\hat{u}^k$ on compacts. Letting $k \rightarrow \infty$ in (1), $(\rho, \hat{u})$ satisfies

$$\rho \hat{u} = \inf_{v \in V_1} [L\hat{u}(x, v) + r(x, v)\hat{u}]. \quad (3)$$

For any admissible control v_t , (2) yields

$$\limsup_{T \rightarrow \infty} \frac{1}{T} E_x [e^{\int_0^T r_k(X_t, v_t) dt}] \geq \rho_k.$$

Therefore using the fact that $r_k \leq r$ and letting $k \rightarrow \infty$ we have

$$\limsup_{T \rightarrow \infty} \frac{1}{T} E_x [e^{\int_0^T r(X_t, v_t) dt}] \geq \rho. \quad (4)$$

Hence $\beta \geq \rho$.

To establish the reverse inequality, let $\hat{u}$ and ρ be as above and v^* be a minimizing measurable selector in (3). By the near monotonicity assumption, $\inf_v r(\cdot, v) > \beta \geq \rho$ outside a compact set C . We know that for some $v > 0$, $\hat{u} \geq v > 0$ in C . Let $x \in C^c$. Let τ_C denote the hitting time to C . By Fatou's lemma,

$$\hat{u}(x) \geq E_x[e^{\int_0^{\tau_C} (r(X_t, v_t^*) - \rho) dt} \hat{u}(X_{\tau_C})] \geq v.$$

This proves that $\hat{u}$ is bounded below by v . Using Ito's formula and Fatou's lemma, we have

$$\hat{u}(x) \geq E_x[e^{\int_0^T (r(X_t, v_t^*) - \rho) dt} \hat{u}(X_T)] \geq v E_x[e^{\int_0^T (r(X_t, v_t^*) - \rho) dt}].$$

Taking log, dividing by T and letting $T \uparrow \infty$ on both sides, we have

$$\limsup_{T \rightarrow \infty} \frac{1}{T} E_x[e^{\int_0^T r(X_t, v_t^*) dt}] \leq \rho. \quad (5)$$

Thus $\beta \leq \rho$. Moreover it follows from (4) and (5) that v^* is optimal for the risk sensitive control problem. This completes the proof of Theorem 2.2.

References

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