Erratum to: Semigroups of partial isometries

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One of the corollaries to the first main result of our paper is not true as it stands: Theorem 3.9 and its proof should be replaced as follows.

Theorem *Let S be a self-adjoint finitely generated semigroup of partial isometries. That is, $S = \langle T_1, T_1^*, \dots, T_n, T_n^* \rangle$ for some $T_1, \dots, T_n$. If $\text{ran } T_i = \text{ran } T_j$, $\ker T_i = \ker T_j$ and $T_i T_j = T_j T_i$ for all i and j , then S is atomic in the sense that the measure space from Theorem 3.1 corresponding to S is purely atomic.*

Proof For each i , let $\mathcal{L}_i$ be the lattice of subspaces of $\mathcal{H}$ generated by the ranges and the kernels of T_i^k , $k = 0, 1, 2, \dots$. Let us show that $\mathcal{L}_i = \mathcal{L}_j$ for all i and j . It is enough to establish that $\text{ran } T_i^k = \text{ran } T_j^k$ and $\ker T_i^k = \ker T_j^k$, $k = 0, 1, 2, \dots$. If $k = 0$, the statement is trivial. Assume by induction that the statement is proved for some k . Pick $x \in \text{ran } T_i^{k+1}$. There is $y \in \mathcal{H}$ such that $x = T_i^{k+1} y = T_i T_i^k y$. Since $\text{ran } T_i^k = \text{ran } T_j^k$, there is $z \in \mathcal{H}$ such that $T_i^k y = T_j^k z$. Then $x = T_i T_j^k z$. By commutativity, $x = T_j^k T_i z$. Since $\text{ran } T_i = \text{ran } T_j$ by the assumptions of the theorem, there is $v \in \mathcal{H}$ such that $x = T_j^k T_j v = T_j^{k+1} v$. So, $\text{ran } T_i^{k+1} = \text{ran } T_j^{k+1}$. Similarly,

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if $x \in \ker T_i^{k+1}$, that is, $T_i^{k+1}x = 0$, then $T_i(T_i^k x) = 0$. Assuming by induction that $\ker T_i^k = \ker T_j^k$, we get $T_i(T_j^k x) = 0$. By commutativity, $T_j^k(T_i x) = 0$. Again, using the assumptions of the theorem, $T_j^k(T_j x) = T_j^{k+1}x = 0$. So, $\ker T_i^{k+1} = \ker T_j^{k+1}$. This shows that $\mathcal{L}_i = \mathcal{L}_j$ for all i and j .

For each $i = 1, \dots, n$, let $\mathcal{S}_i$ denote the singly generated semigroup $\langle T_i, T_i^* \rangle$. By Theorem 2.1, each $\mathcal{S}_i$ is atomic (in the sense of Theorem 3.1). It readily follows from $\mathcal{L}_i = \mathcal{L}_j$ that the atoms corresponding to $\mathcal{S}_i$ are the same as those for $\mathcal{S}_j$ ($i, j = 1, \dots, n$). So, each generator (and, hence, each member) of $\mathcal{S}$, when applied to any of these atoms, turns it either into zero or into another atom. $\square$

Remark There are examples showing that without the range assumption or the kernel assumption the statement is not true even with commuting generators.

There is also a more obvious omission of the word “irreducible” in the statement of Corollary 3.12.