CORRIGENDUM

RGC_n -Commutative Δ -Semigroups

Zhonghao Jiang and Attila Nagy

Communicated by Thomas E. Hall

A semigroup S is called a Δ -semigroup if the lattice of all congruences of S is a chain with respect to inclusion. In [3], A. Nagy determined RGC_n -commutative Δ -semigroups. Unfortunately, the description of semilattice decomposable RGC_n -commutative Δ -semigroups (Theorem 6 of [3]) is incorrect. In fact, Lemma 16 of [3] was used in the proof of Theorem 6 of [3]. But the proof of Lemma 16 of [3] is not complete, because the proof did not cover the case when S_0 does not contain a zero element.

The aim of this note is to give a correct description of semilattice decomposable RGC_n -commutative Δ -semigroups. All terms here are as defined in [3].

We modify Theorem 6 of [3] to the following theorem.

Theorem. *A semilattice decomposable RGC_n -commutative semigroup is a Δ -semigroup if and only if it is isomorphic to either G^0 or R^0 or R^1 or N^1 , where G is a non-trivial subgroup of a quasicyclic p -group (p is a prime), R is a two-element right zero semigroup and N is a commutative nil semigroup whose ideals form a chain with respect to inclusion.*

Proof. Sufficiency is immediate.

Necessity. Assume that S is a semilattice decomposable RGC_n -commutative Δ -semigroup. Then S is a semilattice of archimedean semigroups S_0 and S_1 with $S_0 S_1 \subseteq S_0$. By Theorem 4 and Lemma 12 of [3], we get that S_0 contains an idempotent. Since S is $\mathcal{R}$ -commutative, $S_0 a \subseteq a S_0$ for all $a \in S_0$. By the dual of Lemma 2 of [1], S_0 is an ideal extension of a direct product of a right zero semigroup and a group by a nil semigroup. Then, by Lemma 12 and Lemma 9 of [3], we conclude that S_0 is either a right zero semigroup or a nil semigroup.

Now we consider S_1 . Since S is a Δ -semigroup, so is S_1 . Then, by Theorem 5 of [3] and Theorem 1.57 of [2], S_1 is either a subgroup of a quasicyclic p -group (p is a prime) or a two-element right zero semigroup. There are three cases.

Case 1. If S_0 is a right zero semigroup, then both S_0 and S_1 are exponential [4]. We will show that S is exponential. If S_1 is a right zero semigroup, then S is a band and so S is exponential. If S_1 is a commutative

group, then let e be the identity of S_1 . For every $x \in S_0, y \in S_1$, we have that $y^n xy^{n+1} = y^{n+1} xy^n$, because S is $G C_n$ -commutative. Thus $exy = yxe$ and so $xy = xe$. It follows that, for each positive integer k ,

$$(xy)^k = xy = xe = xy^k = x^k y^k$$

and

$$(yx)^k = yx = x = y^k x^k.$$

Hence S is also exponential. By Theorem 2.7 of [4], either $S = S_0^1$ such that S_0 is a two-element right zero semigroup or $S = S_1^0$ such that S_1 is either a subgroup of a quasicyclic p -group (p is a prime) or a two-element right zero semigroup.

Case 2. If S_0 is a nil semigroup and S_1 is a subgroup of a quasicyclic p -group (p is a prime), then by Theorem 1.59 of [2], either $|S_0| = 1$ or $|S_1| = 1$. It follows that $S = S_0^1$ such that S_0 is a commutative nil Δ -semigroup (hence the ideals of S_0 form a chain with respect to inclusion) or $S = S_1^0$ such that S_1 is a subgroup of a quasicyclic p -group (p is a prime).

Case 3. If S_0 is a nil semigroup and S_1 is a two-element right zero semigroup, then by Lemma 17 of [3], $|S_0| = 1$. Thus $S = S_1^0$ such that S_1 is a two-element right zero semigroup. ■

In the end of [3], the author pointed out that the classes of Δ -semigroups in the class of RGC_n -commutative semigroups and in the class of RC -commutative ones are identical. By the Theorem of this note, we know that the class of RGC_n -commutative Δ -semigroups is different from the class of RC -commutative Δ -semigroups. In fact, we have the following.

Corollary. *A semigroup S is an RGC_n -commutative Δ -semigroup if and only if S is either an RC -commutative Δ -semigroup or a two-element right zero semigroup with an identity adjoined.*

Acknowledgment

The authors would like to thank Professor T.E. Hall for his many helpful suggestions.

References

- [1] Jiang, Z., *LC-commutative permutable semigroups*, Semigroup Forum, **52** (1996), 191–196.
- [2] Nagy, A., “Special Classes of Semigroups,” Kluwer Academic Publishers, Dordrecht/Boston/London, 2001.
- [3] Nagy, A., *RGC_n -commutative Δ -semigroups*, Semigroup Forum, **57** (1998), 92–100.

- [4] Trotter, P.G., *Exponential Δ -semigroups*, Semigroup Forum, **12** (1976), 313–331.

Department of Mathematics
Northern Jiaotong University
Beijing, 100044
People's Republic of China
zhjjiang@center.njtu.edu.cn

Department of Algebra
Institute of Mathematics
Budapest University of Technology
and Economics
1521 Budapest, Pf91., Hungary
nagyat@math.bme.hu

Received April 10, 2002
and in final form June 9, 2003
Online publication July 18, 2003