Erratum to: The spin of prime ideals

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A certain argument concerning the coprimality of two variables turns out to be incorrect. We are grateful to Ruofan Wang for finding this error. On page 728 we claimed that q and m can be taken to be coprime. This is not always true but actually, in the application, we needed only that q does not divide m . As a consequence, all of the results remain unchanged, except for a few modifications in the proofs. We make the following changes.

1. On page 723, line 2, replace $2^{2h+3} f$ by $2^{2h+3} f \mathfrak{D}$, where $\mathfrak{D}$ is the different.

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2. On page 727, replace the first two lines following (6.21) by

... where $\mathfrak{g}$ takes from $\mathfrak{c}$ all prime ideals of degree greater than one (recall that there are no ramified primes in $\mathfrak{c}$ which is coprime to $\mathfrak{D}$), and all primes of degree one for which some different ...

and three lines later, in the congruence, replace “mod q ” by “mod $\mathfrak{q}$ ”.

3. On page 728, replace the text starting from ‘Finally, the congruence ...’ and ending after (6.25), by

Finally, the congruence (6.12), together with $a \equiv a_0 \pmod{g_0}$, means that a runs over a certain arithmetic progression of modulus k which divides $g_0 m F$ where $m = Nm$. Hence, we see that the inner sum in (6.23) can be arranged as the sum of n sums, each of which runs over a single segment of an arithmetic progression of length $\ll x^{1/n}$. Since $\chi_{\mathfrak{q}}(\ell) = (\ell/\mathfrak{q})$ is the real primitive Dirichlet character of modulus $q = N\mathfrak{q}$ (see (3.17) and Lemma 3.6), we have here n incomplete character sums of length $\ll x^{1/n}$ and modulus $q \ll x$. Therefore, Conjecture C_n applies, (or rather its consequence for arithmetic progressions as described at the end of Section 9), giving (see (9.4)),

$$T(x; \beta) \ll g_0 x^{\frac{1-\delta}{n} + \varepsilon} \quad (6.25)$$

except when $q \mid k$. Note that $q \mid k$ implies $q \mid m$.

4. On page 729, replace lines 4–6 by

Recall that (6.25) holds if $q \nmid m$. Hence, because of the factorizations (6.18) and (6.21), we have (6.25) unless the following holds:

$$p \mid N(\beta - \beta') \Rightarrow p^2 \mid mFN(\beta - \beta'). \quad (6.26)$$

5. Accordingly, in the four following displays, replace respectively:

$F^2 N(\beta - \beta')$ squarefull by $p \mid N(\beta - \beta') \Rightarrow p^2 \mid mFN(\beta - \beta')$,

$F^2 N(\gamma)$ squarefull by $p \mid N(\gamma) \Rightarrow p^2 \mid mFN(\gamma)$,

$F^2 N\mathfrak{b}$ squarefull by $p \mid N\mathfrak{b} \Rightarrow p^2 \mid mFN\mathfrak{b}$,

$F^2 b$ squarefull by $p \mid b \Rightarrow p^2 \mid mFb$,

and then, on the line preceding (6.27), before the words “Hence, we conclude”, insert the sentence:

Note that we can assume $m \leq x$ else the sum in (6.1) is empty.

6. On page 730, change the third sentence after (6.33) to the following one:

... The condition for $N(\gamma)$ is now replaced by $g \mid N(\gamma)$.

7. The argument following (9.7) up to the end of Section 9 is correct, but no longer sufficient for the application. Therefore, we replace it by the following modification:

Now, suppose that $\chi = \chi_q$ is real, primitive of odd modulus q . For $n = km + \ell$, since q is necessarily squarefree, we have

$$\chi(n) = \chi_{(q,k)}(\ell) \chi_{q/(q,k)}(k) \chi_{q/(q,k)}(m+r)$$

with $kr \equiv \ell \pmod{q/(q,k)}$. It follows that the sum (9.7) is bounded by $S_{\chi_{q/(q,k)}}(M', N/k)$ with $M' = (M - \ell)k^{-1} + r$. Since the bound (9.4) does not depend on M , it remains valid for $S_\chi(M, N; k, \ell)$ provided $\chi \pmod{q}$ is real, primitive with q odd, $q \nmid k$.