Erratum to: Positivity of relative canonical bundles and applications

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In [2, Theorem II/Theorem 2] we drew a general fact from our arguments that is too strong as it stands. For smooth spaces Y and normal crossings divisors A (or analytic subsets that are locally contained in normal crossings divisors) the original version is correct. It is also correct, if we assume that the given line bundle possesses local extensions. A weaker (sufficient) result is stated below. We would like to thank Sebastien Bouksom for pointing out the inconsistency.

Theorem II' *Let Y be a normal space and $Y' \subset Y$ the complement of a closed analytic, nowhere dense subset. Let L' be a holomorphic line bundle on Y' together with a hermitian metric h' of semi-positive curvature, which may be singular. Assume that the curvature current can be extended to a desingularization of Y as a positive closed current ω . Then there exists a modification $\tau : \tilde{Y} \rightarrow Y$, which is biholomorphic over Y' , and a holomorphic line bundle (L, h) on $\tilde{Y}$ with a singular hermitian metric of semi-positive curvature. The restriction to Y' via τ is isomorphic to (L', h') . The metric h can be chosen with at most analytic singularities, if ω has this property.*

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Proof Let $\mu : Y_{(1)} \rightarrow Y$ be a desingularization and ω on $Y_{(1)}$ like in the assumption. Let $Y' = Y \setminus A$. Set $A_{(1)} = \mu^{-1}(A)$. We take an embedded resolution $\mu_{(1)} : Y_{(2)} \rightarrow Y_{(1)}$ of singularities of $A_{(1)}$ in $Y_{(1)}$ so that $A_{(2)} = \mu_{(1)}^{-1}(A_{(1)})$ only has simple normal crossings singularities. Let $Y'_{(2)} = Y_{(2)} \setminus A_{(2)}$, $\mu_{(2)} = \mu \circ \mu_{(1)}$, and $\mu'_{(2)} = \mu_{(2)}|_{Y'_{(2)}}$. Let $\mathcal{L}'$ be the sheaf induced by L' . We apply our basic result [2, Proposition 6] to $\mu'^*_{(2)}\mathcal{L}'$ together with the pullback of the positive current $\mu^*_{(1)}\omega$: The sheaf $\mu'^*_{(2)}\mathcal{L}'$ can be extended to an invertible sheaf $\mathcal{L}_{(2)}$ on $Y_{(2)}$, which carries an extended hermitian metric $h_{(2)}$.

Consider the coherent sheaf $\mathcal{S} = \mu_{(2)*}(\mathcal{L}_{(2)})$. Since Y is normal, $\mu_{(2)}$ has connected fibers. Hence the sheaf $\mathcal{S}$ coincides with $\mathcal{L}'$ over Y' .

We apply Hironaka's flattening theorem: There exists a sequence of monoidal transformations with centers contained in A resulting in a modification $\tau : \tilde{Y} \rightarrow Y$ such that the pull-back modulo torsion $\mathcal{L}$ of $\mathcal{S}$ is $\mathcal{O}_{\tilde{Y}}$ -flat, i.e. locally free of rank one.

The sheaf $\mathcal{L}$ is equipped with a singular hermitian metric h as follows. Over $\tilde{Y} \setminus \tau^{-1}(A)$ the metric exists by assumption. Since the extended line bundle exists, for coordinate neighborhoods $U \subset \tilde{Y}$ the plurisubharmonic function $-\log h|_{(U \setminus \tau^{-1}A)}$ is well-defined and bounded near $\tau^{-1}A$ so that the extension theorems for plurisubharmonic functions on normal spaces apply.

For singular hermitian metrics on line bundles and their curvature forms on normal complex spaces analytic singularities are defined like in the smooth case. If ω has analytic singularities, so has $\mu^*_{(1)}(\omega)$. Now the curvature of $h_{(2)}$ has at most analytic singularities, since it differs from $\mu^*_{(1)}(\omega)$ by multiples of currents of integration along divisorial components of $\mu^*(A)$ according to [2, Proposition 6, (37)]. This fact translates into analytic singularities for h . $\square$

We mention our application of Theorem II' to the generalized Weil–Petersson form ω^{WP} . We showed in [2, Proposition 9, Theorem 4] that ω^{WP} extends as a positive current with a $\partial\bar{\partial}$ -potential. By the argument of [2, Sect. 7] the curvature form $\omega_{\mathcal{X}}$ of the relative canonical bundle has at most analytic singularities: The lower C^0 -estimates yield $|\sigma(s)|^{2\ell} e^{-u} \leq \tilde{c}$ for some $\ell \in \mathbb{N}$ and some positive constant $\tilde{c}$. Now the proof of [2, Theorem 4] implies that the extension of ω^{WP} has at most analytic singularities.

The gap in the original version (last paragraph of the proof of Theorem 2) arose from the non-existence of a certain determinant line bundle: By a result of Forster and Knorr [1, Theorem II] direct images of complexes that are perfect relative to the base are again perfect, i.e. locally quasi-isomorphic to a bounded complex; in particular a determinant sheaf exists—a result that was being used for families of compact complex manifolds. In the above case we were dealing with a complex, which is perfect but not perfect relative to the base.

We take the opportunity to correct misprints.

Page 50 formula (87) reads $\partial(A_{\bar{s}} \cup \psi^k) = 0$.

Page 51 line +9: replace

$$\langle (G_{\bar{\partial}} \bar{\partial}^* L_{\bar{v}} \psi^k)', \bar{\partial}(L_{\bar{v}} \psi^\ell)' \rangle \quad \text{by} \quad \langle (G_{\bar{\partial}} \bar{\partial}^* L_{\bar{v}} \psi^k)', \bar{\partial}^*(L_{\bar{v}} \psi^\ell)' \rangle.$$

Page 51 line -13: replace $\sum \lambda_v \rho_v$ by $\sum \rho_v$.

Page 52 line +10: replace $\cdots = m \cdot a_{s\bar{\beta}_{n+1}}$ by $\cdots = m \cdot a_{s\bar{\beta}_{n+1}} \psi$.

Page 52 line -6: delete this line.

References

1. Forster, O., Knorr, K.: Relativ-analytische Räume und die Kohärenz von Bildgarben. *Invent. math.* **16**, 113–160 (1972)
2. Schumacher, G.: Positivity of relative canonical bundles and applications. *Invent. math.* **190**, 1–56 (2012)