Erratum: On the Dirichlet problem for minimal graphs in hyperbolic space

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Let $H^{n+1} = \{(x, y) \in \mathbb{R}^n \times \mathbb{R}, y > 0\}$ denote the hyperbolic space equipped with the metric

$$ds^2 = \frac{dx^2 + dy^2}{y^2}.$$

In this model, $H_\infty^{n+1} = \mathbb{R}^n \times \{0\} \cup \{*\}$ is the one-point compactification of $\mathbb{R}^n \times \{0\}$ that can be identified with the asymptotic infinity of H^{n+1} .

Let $N^{n-1} \subset H_\infty^{n+1}$ be a closed embedded $(n-1)$ -dimensional submanifold. Then there exists a complete absolutely area-minimizing integral n -current Σ asymptotic to N^{n-1} at infinity; see, for example [1, 2]. It was also noted in [1, 2] that in the case $n \leq 6$, currents obtained are smoothly embedded complete hypersurfaces. In the case $n \geq 7$, as in the Euclidean case, there can be a singular set of Hausdorff dimension at most $n-7$.

This erratum should have been written nearly twenty years ago [5]. I apologize for any inconvenience or confusion that may have resulted from the lengthy delay in its composition.

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The question of the boundary behavior of such hypersurfaces was discussed in [3]; see also [4] for the case of general higher codimension. It was shown that if N is a C^1 -submanifold of H_∞^{n+1} , then $(\Sigma \cup N) \cap \{y < \rho_N\}$ is a finite union of C^1 -submanifolds with boundary for some $\rho_N \geq 0$. Moreover, these C^1 -submanifolds have disjoint analytic interiors and meet $\mathbb{R}^n \times \{0\}$ orthogonally at N .

Concerning higher-order regularity, the following result was claimed in [5].

Theorem [5] *Suppose that N^{n-1} is $C^{k,\alpha}$ with either $1 \leq k \leq n-1$ and $0 \leq \alpha \leq 1$ or $k \geq n$ and $0 < \alpha < 1$. Then there is a positive ρ_N such that $(\Sigma \cup N) \cap \{y < \rho_N\}$ is a $C^{k,\alpha}$ submanifolds hypersurface with boundary.*

This result is not fully correct. The proofs given in [5] for the case when either $1 \leq k \leq n-1$ and $0 \leq \alpha \leq 1$ or $k = n$ and $0 < \alpha < 1$ were correct. But the proof in [5] for the case $k \geq n+1$ was incorrect; in fact, the result is not true for $k \geq n+1$ when $n \geq 3$ is odd. The correct statements as well as the proofs for the case $k \geq n+1$ were provided in Y. Tonegawa's 1993 Ph.D. thesis at New York University [6], and part of that thesis related to these results appeared in [7].

For the convenience of readers, we summarize the full and correct statements as follows:

Theorem *Suppose that N^{n-1} is of class $C^{k,\alpha}$.*

- (I) *If $1 \leq k \leq n-1$ and $0 \leq \alpha \leq 1$ or $k = n$ and $0 \leq \alpha < 1$, then $M = (\Sigma \cup N) \cap \{y < \rho_N\}$ is a $C^{k,\alpha}$ hypersurface with boundary.*
- (II) *If $k \geq n+1$ and $0 < \alpha < 1$, then,*
 - (IIa) *for n even, M is a $C^{k,\alpha}$ hypersurface with boundary;*
 - (IIb) *for n odd, there is a necessary and sufficient condition on N (a differential equation of order $(n+1)$ to be satisfied by N) so that M will be a $C^{k,\alpha}$ hypersurface with boundary provided this additional condition is satisfied.*

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