

## Erratum to: The scalar curvature flow on $S^n$ —perturbation theorem revisited

Xuezhang Chen · Xingwang Xu

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### Erratum to: Invent Math

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In this erratum, our main purpose is to correct some careless mistake in our paper. The error occurs in the proof of Lemma 4.2. This error effects the size  $\beta$  of initial energy level through the selection of  $\epsilon_0$ . However our main statement, Theorem 1.2, will stand as it is.

The details are as follows.

- (1) Before the statement of Lemma 4.2: the third sentence in the paragraph before the equation (4.2), “Hence we set ... and set ...” should be changed to “Since  $\frac{\max_{S^n} f}{\min_{S^n} f} < \delta_n \leq 2^{\frac{2}{n-2}}$  for all  $n \geq 3$ , choose  $\epsilon_f =$

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X. Chen (✉)

Department of Mathematics & IMS, Nanjing University, Nanjing 210093, P.R. China  
e-mail: [chenxuezhang@sina.com](mailto:chenxuezhang@sina.com)

X. Chen

Beijing International Center for Mathematical Research, Peking University, Beijing 100871, P.R. China

X. Xu

Department of Mathematics, National University of Singapore, Block S17 (SOC1),  
10 Lower Kent Ridge Road, Singapore 119076, Singapore  
e-mail: [matxuxw@nus.edu.sg](mailto:matxuxw@nus.edu.sg)

$\frac{1}{2}[\delta_n(\frac{\min_{S^n} f}{\max_{S^n} f}) - 1] > 0$ . Hence set  $\epsilon_0 = \min\{\frac{1}{2}[2^{(\frac{n-2}{n})^2} - 1], \epsilon_f\} > 0$  if  $n \leq 3 \leq 4$  and  $\epsilon_0 = \min\{\frac{1}{2}(2^{\frac{n-4}{n}} - 1), \epsilon_f\} > 0$  for  $n \geq 5$  and set

$$\beta = n(n-1)(1 + \epsilon_0)^{\frac{n-2}{n}} \left(\min_{S^n} f\right)^{-\frac{n-2}{n}}. \quad (4.2)''$$

(2) In the proof of Lemma 4.2, for the first long equation, it should read as

$$\begin{aligned} L(n(n-1) - \epsilon) &\leq \sum_{j=1}^L \left[ \omega_n^{-1} \int_{B_r(P_j)} |R(g(t_k))|^{n/2} d\mu_{g(t_k)} \right]^{2/n} \\ &\leq L^{1-\frac{2}{n}} \left[ \int_{S^n} |R(g(t_k))|^{n/2} d\mu_{g(t_k)} \right]^{2/n} \\ &\leq L^{1-\frac{2}{n}} \left[ \int_{S^n} |R(g(t_k)) - \alpha(t_k)f|^{n/2} d\mu_{g(t_k)} \right]^{2/n} \\ &\quad + L^{1-\frac{2}{n}} \alpha(t_k) \left[ \int_{S^n} |f|^{n/2} d\mu_{g(t_k)} \right]^{2/n}. \end{aligned}$$

(3) In the proof of Lemma 4.2, for the second long equation, it should display as follows:

$$\begin{aligned} &\alpha(t_k) \left[ \int_{S^n} |f|^{n/2} d\mu_{g(t_k)} \right]^{2/n} \\ &\leq E_f[u_k] \left[ \int_{S^n} |f|^{n/2} d\mu_{g(t_k)} \right]^{2/n} \left[ \int_{S^n} f d\mu_{g(t_k)} \right]^{-2/n} \\ &\leq E_f[u_0] \left[ \max_{S^n} f \right]^{\frac{n-2}{n}} \\ &\leq \beta \left[ \max_{S^n} f \right]^{\frac{n-2}{n}} \\ &\leq n(n-1) \left[ (1 + \epsilon_0) \frac{\max_{S^n} f}{\min_{S^n} f} \right]^{\frac{n-2}{n}} \\ &\leq n(n-1) \left[ (1 + \epsilon_f) \frac{\max_{S^n} f}{\min_{S^n} f} \right]^{\frac{n-2}{n}}. \end{aligned}$$

(4) After the mentioned second long equation: the sentences should be changed as follows: "Let  $k \rightarrow \infty$ , one has  $Ln(n-1) \leq L^{1-\frac{2}{n}}n(n-1)[(1 + \epsilon_f) \frac{\max_{S^n} f}{\min_{S^n} f}]^{\frac{n-2}{n}}$ . With our choices of  $\epsilon_f$ , we get  $L^{\frac{2}{n}} \leq$

$[(1 + \epsilon_f) \frac{\max_{S^n} f}{\min_{S^n} f}]^{\frac{n-2}{n}} < 2^{\frac{2}{n}}$ . One easily concludes that  $L = 1$  since  $L$  is a natural number.”

We also would like to take this opportunity to correct other typo: namely, in the proof of Lemma 3.1, in case (i), after the long equation, the phrase “by observing that  $\frac{\alpha(t)}{E[u]} = \int_{S^3} f u^6 d\mu_{S^3} \leq M \int_{S^3} u_0^6 d\mu_{S^3}$  and ...” should be replaced by “by observing that  $\frac{\alpha(t)}{E[u]} = (\int_{S^3} f u^6 d\mu_{S^3})^{-1} \leq m^{-1} (\int_{S^3} u_0^6 d\mu_{S^3})^{-1}$  and ...”.

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