Erratum

Erratum to: Morrey Potentials and Harmonic Maps

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There are two errors in our original paper:

- $\mathcal{I}_\alpha \mu$ in Theorem 2.4 should be replaced by $I_\alpha \mu$, and then $\approx$ in Theorem 2.4 (i) should be replaced by $\lesssim$.
- The estimate for the Wolff potential in the proof of Theorem 2.5 (plus Theorems 2.7 & 2.9) is defective. So, the main parts of Theorems 2.5, 2.7 & 2.9 should be replaced by

Theorem 0.1. *Suppose μ and f are a non-negative Radon measure and a non-negative function with support in a bounded domain $\Omega \subset \mathbb{R}^N$, respectively. Let*

$$\begin{cases} 1 \leq p < \infty; \\ 0 < N - \alpha p < d \leq N; \\ \|\mu\|_d < \infty; \\ 0 < \|f\|_{L^{p,\lambda}} < \infty. \end{cases}$$

- (i) *If $N \geq \lambda > \alpha p$ & $1 < q < \frac{p(d+\lambda-N)}{\lambda-\alpha p}$, then $\frac{(\int_\Omega |I_\alpha f|^q d\mu)^{\frac{1}{q}}}{\|f\|_{L^{p,\lambda}}} \lesssim \|\mu\|_d$.*
- (ii) *If $N \geq \lambda > \alpha p$ & $1 < \gamma \leq q = \frac{p(d+\lambda-N)}{\lambda-\alpha p}$, then $\int_\Omega \left(\frac{I_\alpha f}{\|f\|_{L^{p,\lambda}}} \right)^q \left(\ln \left(1 + \frac{I_\alpha f}{\|f\|_{L^{p,\lambda}}} \right) \right)^{-\gamma} d\mu \lesssim \|\mu\|_d$.*
- (iii) *If $N \geq \lambda = \alpha p$ & $0 \leq \beta \leq 1$, then there is a constant $c_0 > 0$ such that $\int_\Omega \exp \left(\frac{c I_\alpha f}{\|f\|_{L^{p,\lambda}}} \right)^\beta d\mu \lesssim \|\mu\|_d \forall c \in [0, c_0]$.*

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Proof. This follows from [3, Theorem 3.2]. $\square$

The defectiveness has affected our paper [1]:

- [1, Theorem 3.1] should be replaced by Theorem 0.1.
- According to Theorem 0.1, the condition $N \geq d > \lambda - \alpha p$ & $(\lambda, q) \in (0, N) \times (0, \frac{dp}{\lambda - \alpha p})$ in [1, Theorem 3.2] should be replaced by $N \geq d > N - \alpha p$ & $(\lambda, q) \in (0, N) \times (0, \frac{(d+\lambda-N)p}{\lambda - \alpha p})$. Thus, on [1, page 4811], $\frac{dp}{\lambda - \alpha p}$; $\frac{dp}{\lambda - 2p}$; $\frac{dp}{\lambda - (m+1)p}$ should be respectively replaced by $\frac{(d+\lambda-N)p}{\lambda - \alpha p}$; $\frac{(d+\lambda-N)p}{\lambda - 2p}$; $\frac{(d+\lambda-N)p}{\lambda - (m+1)p}$. Moreover, on [1, page 4812], $q < \frac{dp}{\lambda - \alpha p}$ should be replaced by $q < \frac{(d+\lambda-N)p}{\lambda - \alpha p}$.
- In accordance with Theorem 0.1(iii), $N \geq d > \lambda - \alpha p = 0$ in [1, Theorem 3.6] should be replaced by $N \geq d > N - \alpha p > \lambda - \alpha p = 0$.

Furthermore, this defectiveness has percolated down through our paper [2]:

- Since [2, Lemma 3.5] follows from [1, Theorem 3.1] which has been replaced by Theorem 0.1, [2, Lemma 3.5] should be replaced by the following result:

Let $\Omega \subset \mathbb{R}^n$ be a bounded domain, $0 < \alpha, \lambda < n$ and $E \subseteq \Omega$.

- (i) *If $1 < p < \lambda/\alpha, n - \alpha p < d \leq n$ and $1 < q < p(d + \lambda - n)/(\lambda - \alpha p)$, then*

$$\Lambda_d^{(\infty)}(E) \lesssim (C_\alpha(E; L^{p,\lambda}(\Omega)))^{\frac{q}{p}}.$$

- (ii) *If $1 < p = \lambda/\alpha, n - \alpha p < d \leq n$ and $0 < q \leq 1$, then there is a constant $c > 0$ such that*

$$\Lambda_d^{(\infty)}(E) \lesssim \exp\left(-c(C_\alpha(E; L^{p,\lambda}(\Omega)))^{\frac{q}{p}}\right).$$

- According to Theorem 0.1, the Hausdorff dimension estimates in [2, Theorem 2.1]; [2, Theorem 3.6(i)]; [2, Theorem 3.6(ii)], should be respectively replaced by:

$$\begin{cases} \dim_H(\Sigma_{\hat{p}}(u, \Omega)) \leq n - mp; \\ \dim_H(\Sigma_1(I_\alpha f, \Omega)) \leq n - \alpha p; \\ \dim_H(\Sigma_1(I_\alpha f, \Omega)) \leq n - \alpha p. \end{cases}$$

- The Hausdorff dimension of any local singular set in [2, Theorem 2.2] is at most $n - 2$. This follows from Theorem 0.1(iii).
- On [2, page 1264], $d > \mu - \alpha p$; $0 < q < \frac{p(\mu - \alpha p)}{d}$; $\lambda - \alpha p$ should be replaced by $d > n - \alpha p$; $1 < q < \frac{p(d + \mu - n)}{\mu - \alpha p}$; $n - \alpha p$, respectively.
- On [2, page 1265], $d > \mu - \alpha p > \lambda - \alpha p$ & $\dim_H(\mathcal{T}(I_\alpha f, \Omega)) \leq \lambda - \alpha p$ should be replaced by $d > n - \alpha p$ & $\dim_H(\mathcal{T}(I_\alpha f, \Omega)) \leq n - \alpha p$, respectively.
- On [2, page 1266], $\dim_H(\Sigma_{\hat{p}}(u, \Omega)) \leq np/q$ should be replaced by $\dim_H(\Sigma_{\hat{p}}(u, \Omega)) \leq n - mp$.
- On [2, page 1268], $d > \epsilon + np/q$ & $\dim_H(\Sigma_p(g, \Omega)) \leq np/q$ should be replaced by $d > \epsilon + n - mp$ & $\dim_H(\Sigma_p(g, \Omega)) \leq n - mp$, respectively.

- On [2, pages 1269–1270],

$$\dim_H(\Sigma_2(u, B)) = 0; \quad \dim_H(\Sigma_2(v, \mathbb{B}^n)) = 0; \quad \dim_H(\Sigma_2(u, \Omega)) = 0$$

should be replaced respectively by [due to Theorem 0.1(iii)]

$$\dim_H(\Sigma_2(u, B)) \leq n - 2; \quad \dim_H(\Sigma_2(v, \mathbb{B}^n)) \leq n - 2; \quad \dim_H(\Sigma_2(u, \Omega)) \leq n - 2.$$

Consequently, “Therefore, $\Sigma_2(u, \Omega)$ consists of at most isolated points” on [2, page 1270] and the second part of [2, page 1260: Theorem 2.2] are uncertain under $n \geq 3$.

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References

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