Erratum

Erratum to: Analytic Quasi-Periodic Cocycles with Singularities and the Lyapunov Exponent of Extended Harper's Model[★]

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We correct the statement of Theorem 2.5 in [2], which generalizes Avila's "quantization of the acceleration" [1] from analytic $SL(2, \mathbb{C})$ -cocycles to arbitrary non-singular cocycles. All notation is kept precisely as in the paper.

Correctly stated, the theorem should read as follows:

Theorem 1 (Quantization of the acceleration). *Consider an analytic cocycle (β, D) where β is irrational and $\det D(x)$ is bounded away from zero on a strip $\mathbb{T}_\delta$. For $|\epsilon| \leq \delta$, $\omega(\beta, D; \epsilon) \in \frac{1}{2}\mathbb{Z}$. Moreover, for (non-singular) Jacobi cocycles, $\omega(\beta, A^E; \epsilon) \in \mathbb{Z}$.*

Thus, even though in general the acceleration is only half-integer valued, for Jacobi cocycles¹

$$A^E(x) := \begin{pmatrix} E - v(x) & -\bar{c}(x - \beta) \\ c(x) & 0 \end{pmatrix}, \quad (1)$$

the object of interest in the paper, one still has integer-valued acceleration. In particular, the correction of Theorem 2.5 does not have any effect on the conclusions for extended Harper's model.

¹ We mention that despite the appearance of the complex conjugate in the Jacobi operator, the resulting Jacobi cocycle can still be realized as an analytic $M(2, \mathbb{C})$ -valued function on $\mathbb{T}$: Given $c \in C^\omega(\mathbb{T}; \mathbb{C})$, $c(x) =: C(e^{2\pi i x})$, we simply "re-interpret" the complex-conjugate of c as $\bar{c}(x) := C^\sharp(e^{2\pi i x})$ with $C^\sharp(z) := C(\frac{1}{\bar{z}})$, corresponding to a reflection of C on the unit circle. Note that for $x \in \mathbb{T}$ this has no effect since $\bar{c}(x) = c(x)$. It was pointed out to us by some readers, that our notation $\bar{c}(x)$ may be confusing at first glance, which is why we comment on it here.

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The error resulted from accidentally dropping a factor of $\frac{1}{2}$ on one side of (2.13), which correctly reads

$$\frac{1}{2} \int_{\mathbb{T}} \log |\det(D(x + i\epsilon))| dx = \frac{1}{2} L \left(\beta, \begin{pmatrix} \det D_{\epsilon}(x) & 0 \\ 0 & \det D_{\epsilon}(x) \end{pmatrix} \right). \quad (2)$$

It is shown in Lemma 2.6 of [2] that the integral on the left hand side of (2) takes values in $2\pi\mathbb{Z}$ which, taking into account the factor of $\frac{1}{2}$ in (2), yields $\omega(\beta, D; \epsilon) \in \frac{1}{2} \in \mathbb{Z}$, as now stated in Theorem 1.

Integer-valued acceleration for Jacobi cocycles only boils down to the following observation:

$$\frac{1}{2} \int_{\mathbb{T}} \log |\det A^E(x + i\epsilon)| dx = \frac{1}{2} (I_{\epsilon}(c) + I_{-\epsilon}(c)) = I(c), \quad (3)$$

where $I_{\epsilon}(c) := \int_{\mathbb{T}} \log |c(x + i\epsilon)| dx$. The second equality in (3) uses that by Lemma 2.6 (i),

$$I_{\epsilon}(c) = I(c) + 2\pi\epsilon N, \quad (4)$$

for some $N \in \mathbb{Z}$ and $|\epsilon| \leq \delta$.

More generally, one can claim integer-valued acceleration for any non-singular cocycle (β, D) , where $\sqrt{\det D}$ can be defined as a *one*-periodic holomorphic function as opposed to *two*-periodicity, which always holds due to Lemma 2.3. This is a consequence of (2.12) and the following simple fact which may be considered an amendment to Lemma 2.3:

Fact 1. *Let $f \in C_{\delta}^{\omega}(\mathbb{T}; \mathbb{C})$ with $\min_{|\operatorname{Im} z| \leq \delta} |f(z)| > 0$. If*

$$\frac{1}{2\pi} \left(D_+ \int_{\mathbb{T}} \log |f(x + i\epsilon)| dx \right) \Big|_{\epsilon=0} \in 2\mathbb{Z}, \quad (5)$$

there exists $g \in C_{\delta}^{\omega}(\mathbb{T}; \mathbb{C})$ satisfying $g^2 = f$.

Proof. If f is a trigonometric polynomial, the claim follows directly from the form of g , explicitly constructed in the proof of Lemma 2.3, Step 1, therein. For general analytic f , approximating by trigonometric polynomials $f_n \rightarrow f$ on $\mathbb{T}_{\delta}$, the same arguments as in the proof of Lemma 2.6 (i) imply that $(D_+ \int_{\mathbb{T}} \log |f_n(x + i\epsilon)| dx) \Big|_{\epsilon=0}$ eventually stabilizes to its limit $(D_+ \int_{\mathbb{T}} \log |f(x + i\epsilon)| dx) \Big|_{\epsilon=0}$. Since, $\sqrt{f_n} \rightarrow \sqrt{f}$, we conclude the statement as claimed. $\square$

Finally, for the sake of completeness, we would like to correct two minor misprints. Equation (2.22) of course only holds at $\epsilon = 0$; for $\epsilon \neq 0$, in agreement with (3) and (4),

$$L(\beta, (A_{\epsilon}^E)') = L(\beta, B_{\epsilon}^E) + I_{\epsilon}(c) - I(c) = L(\beta, B_{\epsilon}^E) + 2\pi N\epsilon. \quad (6)$$

Similarly, the second equality in (2.4) only holds at $\epsilon = 0$.

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References

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2. Jitomirskaya, S., Marx, C.A.: Analytic quasi-periodic cocycles with singularities and the Lyapunov Exponent of Extended Harper's Model. *Commun. Math. Phys.* **316**(1), 237–267 (2012)

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