Erratum

Boundary Scattering, Symmetric Spaces and the Principal Chiral Model on the Half-Line

N.J. MacKay, B.J. Short

Department of Mathematics, University of York, York YO10 5DD, U.K.

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In the above article, two points require correction. First, the discussion regarding charge conjugation in Sect. 3.2.2 is unnecessary and may be replaced by an appeal only to crossing unitarity. Consequently (3.16) should now read

$$\begin{aligned} E^\dagger E &= I & \text{and} & & \det E &= 1, \\ \rho(\theta) &= \rho(-\theta^*)^* & \text{and} & & \rho(\theta)\rho(-\theta) &= 1. \end{aligned} \quad (3.16)$$

Second, the crossing unitarity relation

$$K^{ij} \left(\frac{i\pi}{2} - \theta \right) = S_{\bar{j}k}^{il}(2\theta) K^{lk} \left(\frac{i\pi}{2} + \theta \right) \quad (3.17)$$

was incorrectly stated, and should be replaced by

$$K^{ij} \left(\frac{i\pi}{2} - \theta \right) = S_{\bar{j}k}^{i\bar{l}}(2\theta) K^{\bar{l}k} \left(\frac{i\pi}{2} + \theta \right). \quad (3.17)$$

These corrections result in some changes to the allowed K -matrices. We give here only the list of the final boundary S -matrices, which replaces the existing Sect. 4.2:

4.2. The boundary S -matrices. First we give a scalar factor which has altered slightly from the original version and also an additional scalar factor:

$$\begin{aligned} \mu(\theta) &= \frac{1}{4\pi^2 c^2} \\ &\times \frac{\Gamma\left(\frac{-\theta}{2i\pi} + \frac{1}{2} + \frac{1}{2i\pi c}\right) \Gamma\left(\frac{-\theta}{2i\pi} + \frac{1}{2} - \frac{1}{2i\pi c}\right) \Gamma\left(\frac{\theta}{2i\pi} + \frac{1}{2i\pi c}\right) \Gamma\left(\frac{\theta}{2i\pi} - \frac{1}{2i\pi c}\right)}{\Gamma\left(\frac{\theta}{2i\pi} + \frac{1}{2} + \frac{1}{2i\pi c}\right) \Gamma\left(\frac{\theta}{2i\pi} + \frac{1}{2} - \frac{1}{2i\pi c}\right) \Gamma\left(\frac{-\theta}{2i\pi} + 1 + \frac{1}{2i\pi c}\right) \Gamma\left(\frac{-\theta}{2i\pi} + 1 - \frac{1}{2i\pi c}\right)}, \end{aligned}$$

$$\lambda(\theta) = \frac{1}{4\pi^2 c^2} \frac{\Gamma\left(\frac{-\theta}{2i\pi} + \frac{M}{N} + \frac{1}{2i\pi c}\right) \Gamma\left(\frac{-\theta}{2i\pi} + 1 - \frac{M}{N} - \frac{1}{2i\pi c}\right) \Gamma\left(\frac{\theta}{2i\pi} + \frac{1}{2i\pi c}\right) \Gamma\left(\frac{\theta}{2i\pi} - \frac{1}{2i\pi c}\right)}{\Gamma\left(\frac{\theta}{2i\pi} + \frac{M}{N} + \frac{1}{2i\pi c}\right) \Gamma\left(\frac{\theta}{2i\pi} + 1 - \frac{M}{N} - \frac{1}{2i\pi c}\right) \Gamma\left(\frac{-\theta}{2i\pi} + 1 + \frac{1}{2i\pi c}\right) \Gamma\left(\frac{-\theta}{2i\pi} + 1 - \frac{1}{2i\pi c}\right)},$$

in both of which c is a freely varying parameter unless otherwise stated below.

The PCM S -matrices may then be written as follows.

4.2.1. $SU(N)$. We have found two types of boundary S -matrix for $SU(N)$,

$$K_{PCM}(\theta) = -(1-h)_\theta \mu(\theta) \left(\nu(\theta)(I + c\theta E_L) \otimes \nu(\theta)(I + c\theta E_R) \right) \quad (4.7a)$$

(whose $c \rightarrow \infty$ limit:

$$K_{PCM}(\theta) = -(1-h)_\theta \left(\eta(\theta) E_L \otimes \eta(\theta) E_R \right) \quad (4.7)$$

is a valid PCM boundary scattering matrix) where

$$E_{L/R} \in \frac{SU(N)}{S(U(N/2) \times U(N/2))},$$

and

$$K_{PCM}(\theta) = -(1-h)_\theta \lambda(\theta) \left(\nu(\theta)(I + c\theta E_L) \otimes \nu(\theta)(I + c\theta E_R) \right), \quad (4.8)$$

where

$$E_{L/R} \in \frac{SU(N)}{S(U(N-M) \times U(M))}.$$

4.2.2. $SO(N)$. For $SO(N)$ three types have been found,

$$K_{PCM}(\theta) = -\left(\frac{h}{2} + 2\right)_\theta \left(\frac{h}{2} + 1\right)_\theta (1-h)_\theta \mu(\theta) \times \left(\nu(\theta)\epsilon_{3,3}(\theta)(I + c\theta E_L) \otimes \nu(\theta)\epsilon_{3,3}(\theta)(I + c\theta E_R) \right) \quad (4.9a)$$

(whose $c \rightarrow \infty$ limit:

$$K_{PCM}(\theta) = -\left(\frac{h}{2} + 2\right)_\theta \left(\frac{h}{2} + 1\right)_\theta (1-h)_\theta \left(\eta(\theta)\epsilon_{3,3}(\theta) E_L \otimes \eta(\theta)\epsilon_{3,3}(\theta) E_R \right) \quad (4.9)$$

is a valid PCM boundary scattering matrix) where

$$E_{L/R} \in \frac{SO(N)}{U(N/2)} \times \{+1, -1\},$$

$$K_{PCM}(\theta) = -\left(\frac{h}{2} + 2\right)_\theta \left(\frac{h}{2} + 1\right)_\theta (1-h)_\theta \left(\eta(\theta)\epsilon_{3,1}(\theta) E_L \otimes \eta(\theta)\epsilon_{3,1}(\theta) E_R \right), \quad (4.10)$$

where

$$E_{L/R} \in \frac{SO(N)}{S(O(N/2) \times O(N/2))},$$

and

$$\begin{aligned} K_{PCM}(\theta) = & - \left(\frac{h}{2} + 2 \right)_\theta \left(\frac{h}{2} + 1 \right)_\theta (1 - h)_\theta \mu(\theta) \\ & \times \left(v(\theta) \epsilon_{3,1}(\theta) (I + c\theta E_L) \otimes v(\theta) \epsilon_{3,1}(\theta) (I + c\theta E_R) \right), \end{aligned} \quad (4.11)$$

where

$$E_{L/R} \in \frac{SO(N)}{S(O(N-M) \times O(M))} \quad \text{and } c = \frac{2ih}{\pi(2M-N)} \text{ in } \mu(\theta).$$

4.2.3. $Sp(N)$. Three types of $K_{PCM}(\theta)$ have also been found for $Sp(N)$,

$$\begin{aligned} K_{PCM}(\theta) = & - \left(\frac{h}{2} + 2 \right)_\theta \left(\frac{h}{2} + 1 \right)_\theta (1 - h)_\theta \mu(\theta) \\ & \times \left(v(\theta) \epsilon_{3,1}(\theta) (I + c\theta E_L) \otimes v(\theta) \epsilon_{3,1}(\theta) (I + c\theta E_R) \right) \end{aligned} \quad (4.12a)$$

(whose $c \rightarrow \infty$ limit:

$$K_{PCM}(\theta) = - \left(\frac{h}{2} + 2 \right)_\theta \left(\frac{h}{2} + 1 \right)_\theta (1 - h)_\theta \left(\eta(\theta) \epsilon_{3,1}(\theta) E_L \otimes \eta(\theta) \epsilon_{3,1}(\theta) E_R \right) \quad (4.12)$$

is a valid PCM boundary scattering matrix) where

$$E_{L/R} \in \frac{Sp(N)}{U(N/2)},$$

$$K_{PCM}(\theta) = - \left(\frac{h}{2} + 2 \right)_\theta \left(\frac{h}{2} + 1 \right)_\theta (1 - h)_\theta \left(\eta(\theta) \epsilon_{3,3}(\theta) E_L \otimes \eta(\theta) \epsilon_{3,3}(\theta) E_R \right), \quad (4.13)$$

where

$$E_{L/R} \in \frac{Sp(N)}{Sp(N/2) \times Sp(N/2)},$$

and

$$\begin{aligned} K_{PCM}(\theta) = & - \left(\frac{h}{2} + 2 \right)_\theta \left(\frac{h}{2} + 1 \right)_\theta (1 - h)_\theta \mu(\theta) \\ & \times \left(v(\theta) \epsilon_{3,3}(\theta) (I + c\theta E_L) \otimes v(\theta) \epsilon_{3,3}(\theta) (I + c\theta E_R) \right), \end{aligned} \quad (4.14)$$

where

$$E_{L/R} \in \frac{Sp(N)}{Sp(N-M) \times Sp(M)} \quad \text{and } c = \frac{2ih}{\pi(2M-N)} \text{ in } \mu(\theta).$$

4.2.4. *SU(N)-conjugating.* Lastly, we have found two types of representation-conjugating boundary S -matrix for $SU(N)$

$$K_{PCM}(\theta) = \left(\frac{h}{2} + 2\right)_\theta \left(\frac{h}{2} + 1\right)_\theta \left(\epsilon_{1,1}(\theta) E_L \otimes \epsilon_{1,1}(\theta) E_R\right), \quad (4.15)$$

where

$$E_{L/R} \in \frac{SU(N)}{SO(N)},$$

and

$$K_{PCM}(\theta) = \left(\frac{h}{2} + 2\right)_\theta \left(\frac{h}{2} + 1\right)_\theta \left(\epsilon_{1,3}(\theta) E_L \otimes \epsilon_{1,3}(\theta) E_R\right), \quad (4.16)$$

where

$$E_{L/R} \in \{1, \omega^2\} \times \frac{SU(N)}{Sp(N)} \quad (\omega^N = -1).$$

The discussion and conclusions are not significantly affected.

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