Pluriharmonic maps into Kähler symmetric spaces and Sym's formula

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Let M be a simply connected Riemann surface and S^2 the round 2-sphere. A key observation in our paper [4] is a generalization of the following simple fact:

Lemma *A smooth map $h : M \rightarrow S^2$ is harmonic iff the $\mathbb{R}^3$ -valued one-form $\gamma = (h \times h_y)dx - (h \times h_x)dy$ is closed.*

In Theorem 2.2 of our paper [4] we assign this observation to O. Bonnet [2], which is wrong. As explained by Hélein [5], it can be viewed as Noether's theorem for the variational problem for harmonic maps into the sphere, and it was used before in [6, 3, 7] in order to study the corresponding evolution equation. It probably does not go back to the times of Bonnet. However, Bonnet [2] showed that all surfaces of constant mean curvature (those with harmonic Gauss map h) arise up to scaling as $f = g \pm h$ where g is a surface of constant Gaussian curvature. But such surface g is obtained by integrating the above one-form γ , (i.e. $d\gamma = \gamma$), see Equation (27) in our paper [4]. Therefore, we called γ *Bonnet form* and g *Bonnet–Sym–Bobenko map* since Sym [8] and Bobenko [1] have constructed g in a different way. This might be misleading, and we want to point out that there is no doubt about the priority of Sym and Bobenko regarding the construction of g .

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