Erratum

On the embedding problem for infinitely divisible distributions on certain Lie groups with toral center

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It turns out that our paper [1] mentioned in the title has an error. We found recently that the proof of Lemma 3.2 in it implicitly involves a commutation relation which is not valid. At this juncture we do not know whether the main results, Theorem 1.1 and Corollary 1.2, of that paper hold or not. On the other hand, certain weaker statements can be proved along the lines of [1], with small modifications in the arguments. We describe these here briefly.

Let G be a connected Lie group which is a semidirect product $G = NK$, where N is a (connected) nilpotent Lie group such that $\overline{[N, N]}$ (the closure of the commutator subgroup) is compact, and K is a compact group acting on N . Let Ψ be a compact set of probability measures on N such that for every natural number n there exists a probability measure ρ on G such that $\rho^n \in \Psi$; here ρ^n denotes the n -th convolution power of ρ . Then the first (and general) assertion in Theorem 1.1 of [1] is that there exists a $\mu \in \Psi$ which is embeddable on G . This can be shown to hold if it is assumed, additionally, that Ψ is invariant under the conjugation action of N (namely $x\lambda x^{-1} \in \Psi$ for all $\lambda \in \Psi$ and $x \in N$). For a probability measure μ on G which is supported on N and infinitely divisible on G this implies (in the place of the embeddability, in the second assertion there) that there exists a sequence $\{x_i\}$ in N such that, as $i \rightarrow \infty$, $x_i \mu x_i^{-1}$ converge to a probability measure ν which is embeddable on G . Such a result was proved in [4] in the special case of a 4-dimensional group satisfying the condition as above, called the Walnut.

For the groups as in the hypothesis of Corollary 1.2 there, the proof in [1] shows that any measure whose support is not contained in N is strongly root compact,

and hence embeddable whenever it is infinitely divisible. For measures supported on N we only have the weaker assertion as above.

The weaker version of Theorem 1.1 stated above would not suffice where the original statement is used in [2], but it turns out that the proof of the result in [2] can be completed without recourse to Theorem 1.1 of [1]; see [3].

As to the modifications needed to get the above weaker version, we content ourselves by noting that the set C' introduced in the beginning of step (iii) of the proof of the theorem, on page 787, should be replaced by the closure of $\{\xi_j \lambda_j k_j \xi_j^{-1} \mid j \geq m\}$. Under the additional hypothesis as above the elements of the latter set are n -th roots of measures from Ψ . The proof can then be continued with analogous (in fact somewhat simpler) arguments and yields the result mentioned above.

References

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