*Erratum***Estimates for the $\bar{\partial}$ -Neumann problem and nonexistence of C^2 Levi-flat hypersurfaces in $\mathbb{C}P^n$** **Jianguo Cao¹, Mei-Chi Shaw¹, Lihe Wang²**¹ Department of Mathematics, University of Notre Dame, Notre Dame, IN 46556 USA
(e-mail: cao.7@nd.edu/mei-chi.shaw.1@nd.edu)² Department of Mathematics, University of Iowa, Iowa City, IA 52242 USA
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We would like to thank Professors T. Ohsawa and K. Diederich for pointing out an error in our paper [CSW]. The inequality (4.18) in the proof of Proposition 4.5 is not valid. Step 2 in the proof of Proposition 4.5 should be changed to the following stronger statement using the unweighted $\bar{\partial}$ -Neumann operator. Proposition 4.5 still holds after the correction with other steps unchanged.

Step 2. We will extend $\bar{\partial} f_\nu$ to a $(p, 1)$ -form G_ν on the whole $\mathbb{C}P^n$ such that $\bar{\partial} G_\nu = 0$ and $G_\nu \rightarrow 0$ in $L^2_{(p,1)}(\mathbb{C}P^n)$ as $\nu \rightarrow \infty$.

Let $\tilde{g}_\nu$ be a smooth extension of $\bar{\partial} f_\nu$ to $\mathbb{C}P^n$ with

$$\|\tilde{g}_\nu\|_{\mathbb{C}P^n} \leq C \|\bar{\partial} f_\nu\|_{\Omega^+_\nu} \quad (4.15)$$

and

$$\|\bar{\partial} \tilde{g}_\nu\|_{L^2(\Omega^-_\nu, \delta^2_\nu)} \leq C \|\bar{\partial} f_\nu\|_{\Omega^+_\nu}, \quad (4.16)$$

where C is independent of ν . This is possible again using the fact that the boundary is C^2 (For a proof of (4.15)–(4.16), see Lemma 9.3.3 in [CS].)

When $n > 2$, it is obvious that $\bar{\partial} \tilde{g}_\nu$ is $\bar{\partial}$ -closed. When $n = 2$, we can further show that $\bar{\partial} \tilde{g}_\nu$ satisfies the compatibility condition (4.3). In fact, for every $\phi \in \mathcal{C}^1_{(2-p,0)}(\bar{\Omega}^-_\nu) \cap \text{Ker}(\bar{\partial})$, by Stokes' theorem, we have

$$\int_{\Omega^-_\nu} \bar{\partial} \tilde{g}_\nu \wedge \phi = \int_{b\Omega^-_\nu} \tilde{g}_\nu \wedge \phi = \int_{b\Omega^-_\nu} \bar{\partial} f_\nu \wedge \phi = \int_{b\Omega^-_\nu} d(f_\nu \wedge \phi) = 0, \quad (4.17)$$

where the C^1 -smoothness of $b\Omega_v^-$ was used. Since each Ω_v^- is strictly pseudoconvex, hence with Stein neighborhood bases, the space $C^1_{(2-p,0)}(\overline{\Omega_v^-}) \cap \text{Ker}(\bar{\partial})$ is dense in $L^2_{(2-p,0)}(\overline{\Omega_v^-}) \cap \text{Ker}(\bar{\partial})$. This shows that the compatibility condition (4.17) of $\bar{\partial}\tilde{g}_v$ holds for any $\phi \in L^2_{(2-p,0)}(\overline{\Omega_v^-}) \cap \text{Ker}(\bar{\partial})$.

Notice that $\tilde{g}_v$ is not necessarily $\bar{\partial}$ -closed. In what follows, we will correct $\tilde{g}_v$ by some $\{v_v^0\}$ such that v_v^0 has support in Ω_v^- , $\bar{\partial}(\tilde{g}_v - v_v^0) = 0$ and $v_v^0 \rightarrow 0$ in $L^2(\mathbb{C}P^n)$.

Let

$$v_v^0 = -\star \bar{\partial} N_v \star \bar{\partial} \tilde{g}_v$$

be a solution for $\bar{\partial}$ given by the $\bar{\partial}$ -Neumann operator N_v for $(n-p, n-2)$ -forms on Ω_v^- . The solution v_v^0 is in $L^2(\Omega_v^-)$. If we set v_v^0 to be zero outside Ω_v^- , we have from Propositions 4.1 and 4.2,

$$\bar{\partial} v_v^0 = \bar{\partial} \tilde{g}_v \quad \text{in the distribution sense in } \mathbb{C}P^n.$$

We claim that the following uniform estimate holds:

$$\|v_v^0\|_{L^2(\Omega_v^-)} \leq \|\tilde{g}_v\|_{L^2(\Omega_v^-)}. \quad (4.18)$$

We may assume that $\tilde{g}_v$ has compact support in Ω_v^- since compactly supported forms are dense in L^2 . We write

$$v_v^0 = -\star \bar{\partial} N_v \vartheta \star \tilde{g}_v.$$

Notice that $\bar{\partial}\tilde{g}_v = 0$ on the boundary $b\Omega_v$ and $\star \bar{\partial}\tilde{g}_v = -\vartheta \star \tilde{g}_v$ is in $\text{Dom}(\bar{\partial}^*)$. This gives (see e.g. Theorem 3.1.14 in Folland-Kohn [FK] or Theorem 4.4.3 in [CS])

$$\vartheta N \vartheta \star \tilde{g}_v = N \vartheta \vartheta \star \tilde{g}_v = 0.$$

We have

$$\begin{aligned} \|v_v^0\|^2 &= (\bar{\partial} N_v \vartheta \star \tilde{g}_v, \bar{\partial} N_v \vartheta \star \tilde{g}_v) \\ &= (\vartheta \bar{\partial} N_v \vartheta \star \tilde{g}_v, N_v \vartheta \star \tilde{g}_v) \\ &= ((\vartheta \bar{\partial} + \bar{\partial} \vartheta) N_v \vartheta \star \tilde{g}_v, N_v \vartheta \star \tilde{g}_v) \\ &= (\vartheta \star \tilde{g}_v, N_v \vartheta \star \tilde{g}_v) \\ &= (\star \tilde{g}_v, \bar{\partial} N_v \vartheta \star \tilde{g}_v) \\ &\leq \|\tilde{g}_v\| \|v_v^0\|. \end{aligned}$$

This proves the claim (4.18).

Notice that the argument above is similar to the observation that the Bergman projection operator $P = I - \vartheta N \bar{\partial}$ is bounded in L^2 , but one can only do this when $\vartheta \star \tilde{g}_v$ is in the domain of $\bar{\partial}^*$.

Thus there exists $v_v^0 \in L^2(\Omega_v^-)$ with its support contained in $\overline{\Omega_v^-}$ and $\bar{\partial} v_v^0 = \bar{\partial} \tilde{g}_v$ in $\mathbb{C}P^n$. Let $G_v = \tilde{g}_v - v_v^0$. Then G_v is a $\bar{\partial}$ -closed extension of $\bar{\partial} f_v$ to $\mathbb{C}P^n$ and

$$\|G_v\|_{L^2(\mathbb{C}P^n)} \leq \|\tilde{g}_v\|_{L^2(\mathbb{C}P^n)} + \|v_v^0\|_{L^2(\mathbb{C}P^n)} \leq C \|\bar{\partial} f_v\|_{\Omega_v^+}$$

with C independent of v .

References

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Note added in proof:

The first two authors have recently extended the results to Lipschitz Levi-flat hypersurfaces (“Liouville’s Theorem and nonexistence of Lipschitz Levi-flat hypersurfaces in $\mathbb{C}P^n$ ” preprint by J. Cao and M.-C. Shaw).