*Erratum***Integrality of L^2 -Betti numbers****Thomas Schick**

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Abstract. A gap has been found in the proof of Proposition 3 of [3]. Consequently, this result and the consequences drawn in the paper have to be considered as conjectures. In particular, it is not proved that the Atiyah conjecture is stable under extensions with free quotient groups and under free products with free groups.

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The purpose of this note is to announce that the proof of Proposition 3 of [3] has a gap. More precisely, it is not clear that Equation (6) is correct. I thank Pere Ara for his careful reading of the paper and pointing out this mistake. This mistake is parallel to a corresponding gap in the proof of Lemma 11.7 of [2]. The proof of [3, Proposition 3] goes through only for H a finite groups.

It follows that Theorem 1, Corollary 1 and Remark 1 of [3], which are consequences of Proposition 3, have no proof. In particular, the preprint [1] is also withdrawn. Similarly, Theorem 3 and Corollary 3, Proposition 9 and Corollary 6 (all of [3]) have to be modified, as is explained in the following. In [3, Examples 1 and 2], if G is a pure braid group or $G = \mathbb{Z} * (\mathbb{Z} * \mathbb{Z}) \times (\mathbb{Z} * \mathbb{Z})$, then G fulfills the Atiyah conjecture only over $\mathbb{Q}G$. All the other results of the paper [3] remain valid.

The following modified statements are correct:

Definition 1. Let $\mathcal{D}$ be the smallest non-empty class of groups such that

- (1) If $p: G \twoheadrightarrow A$ is a projection of a torsion-free group G onto an elementary amenable group A , and if $p^{-1}(E) \in \mathcal{D}$ for every finite subgroup E of A , then $G \in \mathcal{D}$.
- (2) $\mathcal{D}$ is subgroup closed.
- (3) $\mathcal{D}$ is closed under direct or inverse limits of directed systems.

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With this definition, the following version of Theorem 3 and Corollary 3 is true:

Theorem 1. *If $G \in \mathcal{D}$ then the strong Atiyah conjecture is true for $\mathbb{Q}G$. In particular, $\mathbb{Q}G$ does not contain non-trivial zero-divisors.*

We still have (similar to [3, Proposition 3])

Proposition 1. *The class $\mathcal{D}$ is closed under direct sums, direct products, and free products.*

Every residually torsion-free solvable, and more generally every residually torsion-free elementary amenable group belongs to $\mathcal{D}$.

Moreover, $\mathcal{D}$ contains all torsion-free elements of the classes $\mathcal{C}$ and $\mathcal{C}'$ defined by Linnell.

In particular, the Atiyah conjecture is true over $\mathbb{Q}G$ if G is a direct sum of free groups. This is important, since the proof of the Atiyah conjecture for $\mathbb{C}G$ ($G \in \mathcal{C}'$) given in [2, Section 12 and 13] relies on [2, Lemma 11.7] and therefore is not valid. However, using the weaker result we have established, the methods of [2, Section 13] still apply to give the Atiyah conjecture for $\mathbb{Q}G$, if $G \in \mathcal{C}'$ and there is a bound on the orders of the finite subgroups of G .

The proofs of the above results about the class $\mathcal{D}$ can be given in the same way as the (attempted) proofs of the corresponding results in [3], observing that with the new definition of $\mathcal{D}$ we never have to use those statements which don't have a valid proof.

A corrected version of the paper can be found on the internet at arXiv.org/abs/math.GT/0001101.

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