Erratum
Erratum to: Existence and Strong
Pre-compactness Properties for Entropy
Solutions of a First-Order Quasilinear
Equation with Discontinuous Flux

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There is some inaccuracy in the proofs of main Theorems 1 and 2.

First, in the end of the proof of Theorem 1 we state that passing to the limit as $r \rightarrow \infty$ in relation (5) with $u = u_r$, one can conclude that the limit function $u = u(x)$ is an entropy solution of (1). To justify the passage to the limit, however, we need the strong convergence $\text{sign}(u_r - p) \rightarrow \text{sign}(u - p)$ as $r \rightarrow \infty$, which is not generally valid for arbitrary p . Observe that this strong convergence holds for $p \in P$ where the set P consists of such p that the level set $u^{-1}(p)$ has zero Lebesgue measure. Evidently, $\text{sign}(u_r - p) \rightarrow \text{sign}(u - p)$ as $r \rightarrow \infty$ almost everywhere on Ω for each $p \in P$.

Second, in the end of the proof of Theorem 2 the passage to the limit as $r \rightarrow \infty$ in the sum of relations (71) with $p = p_r^-$ and $p = p_r^+$ is not correct because the functions $\omega_p(x)$ and the measures $|\gamma_p^s|$ are not supposed to be continuous with respect to p . Again, we can only conclude that (71) is satisfied for $p \in P$.

Observe that the indicated above set P has full Lebesgue measure. Therefore, both the mistakes will disappear if we modify slightly the definition of the entropy solution. Namely, one should require that condition (5) holds for $p \in \mathcal{P}$ where $\mathcal{P}$ is some set of full Lebesgue measure (possibly dependent on a solution u), cf. [1]. Moreover, we may assume that requirement (3) is satisfied only for $p \in \mathcal{P}_0$ where $\mathcal{P}_0$ is a set of full measure.

After this improvement, relation (56) in Lemma 5 holds for $a, b \in \mathcal{P}$. This implies that condition (54) is satisfied also for $a, b \in \mathcal{P}$. But, since the sequence $\text{div}_x \varphi(x, s_{a,b}(u_k))$ is uniformly continuous in $H_{\text{loc}}^{-1}(\Omega)$ with respect to a, b while the set $\mathcal{P}$ is dense, we claim that actually (54) is satisfied for all $a, b \in \mathbb{R}$, $a < b$ and the statement of Theorem 6 follows. This allows us to conclude the proofs of

main Theorems 1 and 2 as in the original paper. Finally, the last part in the proof of Theorem 2 containing justification of relation (71) for an arbitrary $p \in \mathbb{R}$ should be removed.

Note that the presented results are covered by the results of recent paper [1], where the more general case of ultra-parabolic equations

$$\operatorname{div} \varphi(x, u) - D^2 \cdot B(x, u) + \psi(x, u) = 0$$

is considered.

Reference

1. PANOV, E.YU.: Ultra-parabolic equations with rough coefficients. Entropy solutions and strong precompactness property. *J. Math. Sci.* **159**(2), 180–228 (2009)

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