The simplified method for transient thermal field analysis in a polymeric DC cable

Jerzy Gołębiowski · Marek Zaręba

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Abstract The simplified method of analysis of the thermal field in the futuristic model of a DC cable is presented. The thermal conductivity of a non-conducting layer is very small comparing with the same parameter of conducting regions. The above causes very fast heat propagation in the core and coating, which in the consequences are approximated by inert elements of the first order. Spatial changes in the field in insulation cannot be neglected, because of the significantly slow heat transfer. For that reason, insulation is treated as an element of distributed parameters. The boundary-initial parabolic problem of a non-conducting region is solved by means of Duhamel's theorem. The fundamental solutions of superposition integrals are determined by the separation of variables method. The maximal deviation of results obtained by the finite element method does not exceed 7.2%. The heating curves and results of verification are presented in a graphic form.

Keywords Transient heat transfer in DC cables · Current transmission · Asymptotic and analytical methods · Duhamel's integral

1 Introduction

Power transmission DC systems play nowadays more important role. They have some advantages (power losses and voltage drops are less than in analogical AC systems). It is predicted that such systems will have much more importance in the future. Technological development will enable manufacturing of conducting polymers and cables could be

moulded of them [1,2]. The thermal field analysis in such systems is therefore of a great importance.

The paper refers to the recent publication of the authors [3]. In Gołębiowski and Zaręba [3], the analytical–numerical method the transient thermal field computation in a polymeric cable was presented. The method relied on the analytical determination of field functions and numerical evaluation of the coefficients of those functions. However, the method presented in Gołębiowski and Zaręba [3] appeared to be rather complex. The three series of eigenvalues of the boundary-initial problem have to be determined, what required development of the complex algorithm. The non-orthogonal systems of functions emerged, as well. A simplified method of the solution of the problem presented in Gołębiowski and Zaręba [3] is discussed in the present paper. The method bases on decomposition of the cable regions. The advantages of Duhamel's theorem [4,5] and superposition principle are utilised.

The subject of analysis is the model of a cable presented in Fig. 1. The internal region (index 1) is the core made of polymer of the large conductivity (close to the conductivity of copper). The middle zone (index 2) is insulation of a large resistivity and electrical strength. The outer region (index 3) is an earthed layer of conducting polymer [1,2]. In the presented model, it is assumed that material parameters are constant and averaged in the range of the rated temperature changes (i.e. from 20–25°C to 70–90°C). The length of a cable is assumed to be much larger than its diameter.

2 The thermal field of a core and outer coating

It was proved in Gołębiowski and Zaręba [3] that the thermal field in the region of the core and external coating is practically uniform at the given instant. It is physically motivated

J. Gołębiowski · M. Zaręba (✉)
Faculty of Electrical Engineering, Technical University of Białystok,
ul. Wiejska 45 D, 15-351 Białystok, Poland
e-mail: m.zareba@pb.edu.pl

by a large value of the thermal conductivity of the core and coating with respect to the analogous parameter of insulation (several hundred times more). In the consequences, it can be assumed, that after switching on of the power supply the whole volume of the region 1 is heating up identically. It also concerns the third material zone. Then, the field in the core and coating depends on time only and practically it is not the function of spatial coordinates. Regions 1 and 3 can be therefore treated as elements of the lumped heat capacitance. It is also indicated by a small value of Biot's number [6] ($Bi = \varepsilon d/\lambda$, where ε coefficient of heat transfer, d layer thickness, λ thermal conductance of the region). In accordance with the above, the differential equation of a thermal balance is described by the following relation [6]

$$\tau_i \frac{dv_i(t)}{dt} + v_i(t) = N_i \quad \text{for } t > 0, R_{i-1} \leq r \leq R_i \quad (1)$$

where $i = 1$ and $R_0 = 0$ (for the core), $i = 3$ (for the coating), τ_1, τ_3 thermal time constants of the core and coating, $v_i(t) = T_i(t) - T_a$ increase in the temperature at instant t for $i = 1$ or $i = 3$, $T_i(t)$ temperature profile in the core ($i = 1$) or coating ($i = 3$), t time, T_a ambient temperature, $N_1 = T_{ll} - T_a$, T_{ll} maximum sustained temperature of the middle insulation, N_3 maximal increase of the temperature in the coating of a cable.

The solution of Eq. (1) with initial condition $v_i(t=0) = 0$ is

$$v_i(t) = v_i(r, t) = N_i \left(1 - e^{-\frac{t}{\tau_i}}\right) \quad \text{for } t > 0, R_{i-1} \leq r \leq R_i \quad (2)$$

where $i = 1$ and $R_0 = 0$ or $i = 3$, $v_i(r, t) = T_i(r, t) - T_a$ spatial-temporal increase in the temperature for $i = 1$ or $i = 3$, r radial coordinate.

The value of increase N_1 for $i = 1$ in Eqs. (1–2) results from the strength of insulation (and is known for the given type of material). The increase N_3 is determined from the thermal Ohm's law [7] for example

$$N_3 = N_1 - R_h I_{ll}^2 \rho \frac{l}{\pi R_1^2}, \quad \text{where} \quad (3)$$

$R_h = \frac{1}{2\pi\lambda_2 l} \ln \frac{R_2}{R_1}$ thermal resistance of insulation [7], $\rho \frac{l}{\pi R_1^2}$ electrical resistance of a core,

$$I_{ll} = \pi R_1 \sqrt{\frac{2\lambda_2\lambda_3\varepsilon R_3(T_{ll} - T_a)}{\rho[\lambda_3\varepsilon R_3 \ln(R_2/R_1) + \lambda_2\varepsilon R_3 \ln(R_3/R_2) + \lambda_2\lambda_3]}} \quad \text{steady state current rating [3],} \quad (4)$$

λ_i thermal conductivity of i th zone, R_i outer radius of i th zone, l cable length ($l \gg 2R_3$), ρ resistivity of a core, ε total heat transfer coefficient. Thermal time constants τ_i for $i = 1$ or $i = 3$ are determined by measurement methods

[8] or on the basis of formula (20a, c) given in Gołębowski and Zaręba [3].

3 Increments of the thermal field of insulation

The next selected subregion of the model (Fig. 1) is the ring of insulation ($R_1 \leq r \leq R_2$). It was proved in Gołębowski and Zaręba [3] that its thermal field is not only the function of time, but depends on the radial coordinate, as well. For that reason, the layer of insulation should be treated as the element of distributed parameters type continuum. The respective boundary-initial problem is formulated with respect to the thermal increases $v_2(r, t)$ referred to the ambient temperature T_a

$$v_2(r, t) = T_2(r, t) - T_a. \quad (5)$$

The increase $v_2(r, t)$ is described by the homogeneous equation of heat conduction [5, 6, 9]

$$\frac{\partial^2 v_2(r, t)}{\partial r^2} + \frac{1}{r} \frac{\partial v_2(r, t)}{\partial r} - \frac{1}{\chi_2} \frac{\partial v_2(r, t)}{\partial t} = 0 \quad \text{for } t > 0 \quad \text{and} \quad R_1 \leq r \leq R_2 \quad (6)$$

where $\chi_2 = \lambda_2/(c_2\delta_2)$ diffusivity, c_2 specific heat, δ_2 mass density of insulation. Before the power supply switching on the system remained at the ambient temperature. Then,

Fig. 1 Cross-section of the model of a cable: $A \left(0 \leq \frac{r}{R_1} \leq 1\right)$; $B \left(\frac{r}{R_1} = \frac{3R_1+R_2}{4R_1} = 1.179\right)$; $C \left(\frac{r}{R_1} = \frac{R_1+R_2}{2R_1} = 1.357\right)$; $D \left(\frac{r}{R_1} = \frac{R_1+3R_2}{4R_1} = 1.536\right)$; $E \left(1.714 = \frac{R_2}{R_1} \leq \frac{r}{R_1} \leq \frac{R_3}{R_1} = 1.969\right)$

from (5), a zero initial condition follows

$$v_2(r, 0) = 0 \quad \text{for } R_1 \leq r \leq R_2. \quad (7)$$

Increases must be continuous on boundary circles ($r = R_1$ and $r = R_2$) because polymer layers strictly adhere each other. The boundary conditions are obtained taking advantage of (2)

$$v_1(R_1, t) = v_2(R_1, t) = N_1 \left(1 - e^{-\frac{t}{\tau_1}}\right) \quad \text{for } t > 0, \quad (8)$$

$$v_3(R_2, t) = v_2(R_2, t) = N_3 \left(1 - e^{-\frac{t}{\tau_3}}\right) \quad \text{for } t > 0. \quad (9)$$

The boundary-initial problem (6–9) presented in the above is linear. Therefore, superposition principle can be applied for its solution. The respective distribution is shown in Fig. 2. In accordance with Fig. 2, the temperature increase in insulation is a sum of the components A and B. In system A, the external boundary condition (for $r = R_2$) was setup to zero, while the internal one (for $r = R_1$) remained unchanged. And vice versa in system B. In the result, it was obtained

$$v_2(r, t) = v_2^{(A)}(r, t) + v_2^{(B)}(r, t). \quad (10)$$

It is convenient to determine the components $v_2^{(A)}(r, t)$, $v_2^{(B)}(r, t)$ applying Duhamel's theorem [4, 5]. It is particularly useful for zero initial condition (7). Besides, boundary conditions (8–9) do not depend on spatial coordinates. In such conditions (for particular components) Duhamel's theorem is simplified to the following form

$$v_2^{(A)}(r, t) = \frac{1}{N_1} \frac{\partial}{\partial t} \int_0^t h_2^{(A)}(r, \zeta) v_2(R_1, t - \zeta) d\zeta \quad \text{for } R_1 \leq r \leq R_2, \quad t > 0, \quad (11)$$

$$v_2^{(B)}(r, t) = \frac{1}{N_3} \frac{\partial}{\partial t} \int_0^t h_2^{(B)}(r, \zeta) v_2(R_2, t - \zeta) d\zeta \quad \text{for } R_1 \leq r \leq R_2, \quad t > 0, \quad (12)$$

where $h_2^{(A)}(r, t)$, $h_2^{(B)}(r, t)$ time-step response of a ring defined in the next chapter, ζ apparent variable of integration. The right sides of Eqs. (11–12) are divided by the amplifications N_1 and N_3 , respectively. They are considered in the step characteristics $h_2^{(A)}(r, t)$, $h_2^{(B)}(r, t)$.

4 Fundamental solutions and the total thermal field

Boundary-initial problems of the systems A and B (Fig. 2) are almost the same. Then in the paper continuation only the system B is analysed more precisely.

As it follows from (12), knowledge of the fundamental solution [in this case the step characteristic $h_2^{(B)}(r, t)$] is necessary for determination of the field with boundary

condition (9). It is described by the boundary-initial problem similar to (6–9). Namely, in (6–7) $v_2(r, t)$ has to be exchanged by $h_2^{(B)}(r, t)$, what is denoted by the symbol $v_2(r, t) \rightarrow h_2^{(B)}(r, t)$. Boundary conditions will change in the significant way

$$h_2^{(B)}(R_1, t) = 0 \quad \text{for } t > 0, \quad (13)$$

$$h_2^{(B)}(R_2, t) = N_3 \quad \text{for } t > 0. \quad (14)$$

The step response $h_2^{(B)}(r, t)$ is investigated by the method of states superposition

$$h_2^{(B)}(r, t) = h_{2s}^{(B)}(r) + h_{2t}^{(B)}(r, t), \quad (15)$$

where $h_{2s}^{(B)}(r)$ steady component ($h_{2s}^{(B)}(r) = \lim_{t \rightarrow \infty} h_2^{(B)}(r, t)$), $h_{2t}^{(B)}(r, t)$ transient component ($\lim_{t \rightarrow \infty} h_{2t}^{(B)}(r, t) = 0$). After formulation of the boundary problem for the steady component $h_{2s}^{(B)}(r)$ and its solution, the stationary component of the system is obtained

$$h_{2s}^{(B)}(r) = \frac{-N_3 \ln \frac{R_1}{r}}{\ln \frac{R_2}{R_1}} \quad \text{for } R_1 \leq r \leq R_2. \quad (16)$$

The partial differential equation for the transient component was obtained exchanging function $v_2(r, t) \rightarrow h_{2t}^{(B)}(r, t)$ in (6). Taking advantage of (13), (15), (16) for $r = R_1$ and of (14), (15) (16) for $r = R_2$ boundary conditions of the investigated solution $h_{2t}^{(B)}(r, t)$ were formulated

$$h_{2t}^{(B)}(R_1, t) = 0. \quad (17)$$

$$h_{2t}^{(B)}(R_2, t) = 0. \quad (18)$$

From zero initial condition for the total step response and from relation (15) follows the initial condition for the transient component

$$h_{2t}^{(B)}(r, 0) = -h_{2s}^{(B)}(r) \quad \text{for } R_1 \leq r \leq R_2, \quad \text{where } h_{2s}^{(B)}(r) \text{ was determined by Eq. (16).} \quad (19)$$

The separation of variables method [10] was applied for the solution of the above boundary-initial problem. Boundary condition (17) was applied to reduce the number of constants. It was finally obtained

$$h_{2t}^{(B)}(r, t) = \sum_{n=1}^{\infty} F_n^{(B)} Z_0 \left(\alpha_n \frac{r}{R_1} \right) e^{-\chi_2 \alpha_n^2 \frac{t}{R_1^2}} \quad \text{for } R_1 \leq r \leq R_2 \quad \text{and } t > 0, \quad \text{where} \quad (20)$$

$$Z_k \left(\alpha_n \frac{r}{R_1} \right) = J_k \left(\alpha_n \frac{r}{R_1} \right) Y_0(\alpha_n) - Y_k \left(\alpha_n \frac{r}{R_1} \right) J_0(\alpha_n) \quad \text{for } k = 0 \quad \text{or } k = 1, \quad (21)$$

α_n eigenvalues of the boundary-initial problem (i.e. constant of separation multiplied by R_1),

$F_n^{(B)}$ coefficients of the field function,

Fig. 2 Superposition of the thermal field in a ring of insulation

$J_k(\dots)$, $Y_k(\dots)$ Bessel's and Neumann's functions of the k th order.

Substitution of (20) to (18) leads to the equation of eigenvalues

$$Z_0\left(\alpha_n \frac{R_2}{R_1}\right) = 0, \quad (22)$$

what unequivocally determines α_n . To compute coefficients $F_n^{(B)}$, (20) was substituted to (19)

$$\sum_{n=1}^{\infty} F_n^{(B)} Z_0\left(\alpha_n \frac{r}{R_1}\right) = -h_{2s}^{(B)}(r). \quad (23)$$

Equation (23) was multiplied by $r Z_0\left(\alpha_m \frac{r}{R_1}\right)$ and integrated by sides in the interval $\langle R_1, R_2 \rangle$. It was obtained in the result

$$\sum_{n=1}^{\infty} F_n^{(B)} I(n, m) = P(m), \text{ where :} \quad (24)$$

$$I(n, m) = \int_{R_1}^{R_2} r Z_0\left(\alpha_n \frac{r}{R_1}\right) Z_0\left(\alpha_m \frac{r}{R_1}\right) dr \quad (25)$$

$$= \begin{cases} 0 & \text{for } n \neq m \\ \|N(m)\|^2 = 0.5 R_2^2 Z_1^2\left(\alpha_m \frac{R_2}{R_1}\right) - 0.5 R_1^2 Z_1^2(\alpha_m) & \text{for } n = m, \end{cases}$$

$$P(m) = - \int_{R_1}^{R_2} r h_{2s}^{(B)}(r) Z_0\left(\alpha_m \frac{r}{R_1}\right) dr = - \frac{N_3 R_1 R_2}{\alpha_m} Z_1\left(\alpha_m \frac{R_2}{R_1}\right). \quad (26)$$

In computing (25–26) relations given in [11], equation of eigenvalues (22) and the equation

$$Z_0(\alpha_n) = Z_0(\alpha_m) = 0, \quad (27)$$

were utilized, which results from (21). Coefficients of the field function were determined from (24) and (25–26)

$$F_n^{(B)} = \frac{P(n)}{\|N(n)\|^2} = \frac{2 N_3 R_1 R_2 Z_1\left(\alpha_n \frac{R_2}{R_1}\right)}{\alpha_n \left[R_1^2 Z_1^2(\alpha_n) - R_2^2 Z_1^2\left(\alpha_n \frac{R_2}{R_1}\right) \right]}, \quad (28)$$

where it was returned to the index n , but $Z_1\left(\alpha_n \frac{R_2}{R_1}\right)$ is computed from (21) for $k = 1$ and $r = R_2$. The fundamental solution present in integral (12) was obtained from combination of (15), (16), (20) and (28)

$$h_2^{(B)}(r, t) = h_{2s}^{(B)}(r) + \sum_{n=1}^{\infty} F_n^{(B)} Z_0\left(\alpha_n \frac{r}{R_1}\right) e^{-\chi_2 \alpha_n^2 \frac{t}{R_1^2}} \quad (29)$$

for $R_1 \leq r \leq R_2$ and $t > 0$.

Then variables in Eqs. (9), (29) were changed ($t \rightarrow t - \zeta$, $t \rightarrow \zeta$). Such modified (9), (29) were substituted to (12).

After integration in ζ , differentiation with respect to t and rearrangement, the investigated temperature increase in the system B (Fig. 2) was determined

$$v_2^{(B)}(r, t) = h_{2s}^{(B)}(r) \left(1 - e^{-\frac{t}{\tau_3}}\right) + \sum_{n=1}^{\infty} F_n^{(B)} \frac{R_1^2 Z_0 \left(\alpha_n \frac{r}{R_1}\right)}{(R_1^2 - \chi_2 \alpha_n^2 \tau_3)} \left(e^{-\chi_2 \alpha_n^2 \frac{t}{R_1^2}} - e^{-\frac{t}{\tau_3}}\right) \quad (30)$$

for $R_1 \leq r \leq R_2, t > 0$.

The step response $h_2^{(A)}(r, t)$ and exponential one $v_2^{(A)}(r, t)$ of the system A were determined in the analogical way (Fig. 2)

$$h_2^{(A)}(r, t) = h_{2s}^{(A)}(r) + \sum_{n=1}^{\infty} F_n^{(A)} Z_0 \left(\alpha_n \frac{r}{R_1}\right) e^{-\chi_2 \alpha_n^2 \frac{t}{R_1^2}} \quad (31)$$

for $R_1 \leq r \leq R_2$ and $t > 0$,

$$v_2^{(A)}(r, t) = h_{2s}^{(A)}(r) \left(1 - e^{-\frac{t}{\tau_1}}\right) + \sum_{n=1}^{\infty} F_n^{(A)} \frac{R_1^2 Z_0 \left(\alpha_n \frac{r}{R_1}\right)}{(R_1^2 - \chi_2 \alpha_n^2 \tau_1)} \left(e^{-\chi_2 \alpha_n^2 \frac{t}{R_1^2}} - e^{-\frac{t}{\tau_1}}\right), \quad (32)$$

for $R_1 \leq r \leq R_2, t > 0$, where

$$h_{2s}^{(A)}(r) = \frac{N_1 \ln \frac{R_2}{r}}{\ln \frac{R_2}{R_1}}, \quad (33)$$

$$F_n^{(A)} = \frac{2N_1 R_1^2 Z_1(\alpha_n)}{\alpha_n \left[R_2^2 Z_1^2(\alpha_n \frac{R_2}{R_1}) - R_1^2 Z_1^2(\alpha_n) \right]}, \quad (34)$$

$Z_k(\dots)$ is defined by the function (21), whereas eigenvalues α_n fulfill Eq. (22).

The investigated temperature distribution in insulation results from (10) and (5)

$$T_2(r, t) = T_a + v_2^{(A)}(r, t) + v_2^{(B)}(r, t) \quad (35)$$

for $R_1 \leq r \leq R_2$ and $t \geq 0$.

It can be noticed that for $t \rightarrow \infty$ Eq. (35) describes the stationary temperature distribution $T_2(r, t \rightarrow \infty) = T_a + h_{2s}^{(A)}(r) + h_{2s}^{(B)}(r)$.

5 Computational examples

A computer program was developed in the Mathematica 6.0 environment [12] based on the presented method. The program computes: eigenvalues from Eq. (22), coefficients of the field function (28), (34), fundamental solutions (29), (31), spatial-temporal temperature distribution (35) and visualizes results. The computations were realised on the computer with dual core processor type Intel Core 2 Duo/3 GHz.

Fig. 3 Fundamental solution (31) at the selected points of the cable cross section: A $\left(0 \leq \frac{r}{R_1} \leq 1\right)$; B $\left(\frac{r}{R_1} = \frac{3R_1+R_2}{4R_1} = 1.179\right)$; C $\left(\frac{r}{R_1} = \frac{R_1+R_2}{2R_1} = 1.357\right)$; D $\left(\frac{r}{R_1} = \frac{R_1+3R_2}{4R_1} = 1.536\right)$; E $\left(1.714 = \frac{R_2}{R_1} \leq \frac{r}{R_1} \leq \frac{R_3}{R_1} = 1.969\right)$

For example the model of a cable [3] was analysed of the core cross-section $S \cong 300\text{mm}^2$. The following data were assumed (Fig. 1):

$$\begin{aligned} R_1 &= 0.0098 \text{ m}, R_2 = 0.0168 \text{ m}, R_3 = 0.0193 \text{ m}, \\ c_1 &= 1000 \text{ J/(kg K)}, \delta_1 = 3680 \text{ kg/m}^3, \lambda_1 = 135 \text{ W/(mK)}, \\ c_2 &= 2000 \text{ J/(kg K)}, \delta_2 = 920 \text{ kg/m}^3, \lambda_2 = 0.27 \text{ W/(mK)}, \\ c_3 &= 1500 \text{ J/(kg K)}, \delta_3 = 1840 \text{ kg/m}^3, \lambda_3 = 94.5 \text{ W/(mK)}, \\ \rho &= 2.25 \cdot 10^{-8} \Omega\text{m}, \end{aligned} \quad (36)$$

$$\varepsilon = 12.7 \text{ W/(m}^2\text{K)}, \quad T_a = 25^\circ\text{C}, \quad T_{ll} = 70^\circ\text{C},$$

where c_i specific heat, δ_i mass density of i th zone. Based on (4), (3) [3] and (36) the following secondary parameters have been computed: $I_{ll} = 789.8 \text{ A}$, $N_1 = T_{ll} - T_a = 45^\circ\text{C}$, $N_3 = 30.54^\circ\text{C}$, $\tau_1 = 2218 \text{ s}$, $\tau_3 = 2349 \text{ s}$.

The series appearing in (29), (31), (35) were found to be fast convergent. To estimate their convergence a ratio of the sum of module of the last ten terms to modulus of the global sum was investigated. It was found, that taking into account 35 terms of any of sums (29), (31), (35) for $t \geq 5 \text{ s}$ with $r = (R_1 + R_2)/2$ the mentioned ratio is less than 10^{-4} . The considered ratio at other points of the cable does not exceed the value of 10^{-3} . Then small values of the investigated ratio indicate insignificant change of modulus of the global sum in summation of the successive terms of series.

The results are presented in a graphic form. Fundamental solutions (29), (31) are drawn in Figs 3 and 4, whereas the investigated temperature profiles at the selected points of a cable—in Fig. 5.

The developed method was verified, as well. For this purpose obtained results were compared with computations made by the finite element method [13]. It was resigned of

Fig. 4 Fundamental solution (29) at the selected points of the cable cross section: A $\left(0 \leq \frac{r}{R_1} \leq 1\right)$; B $\left(\frac{r}{R_1} = \frac{3R_1+R_2}{4R_1} = 1.179\right)$; C $\left(\frac{r}{R_1} = \frac{R_1+R_2}{2R_1} = 1.357\right)$; D $\left(\frac{r}{R_1} = \frac{R_1+3R_2}{4R_1} = 1.536\right)$; E $\left(1.714 = \frac{R_2}{R_1} \leq \frac{r}{R_1} \leq \frac{R_3}{R_1} = 1.969\right)$

Fig. 5 Heating curves (35) at the selected points of the cable cross section: A $\left(0 \leq \frac{r}{R_1} \leq 1\right)$; B $\left(\frac{r}{R_1} = \frac{3R_1+R_2}{4R_1} = 1.179\right)$; C $\left(\frac{r}{R_1} = \frac{R_1+R_2}{2R_1} = 1.357\right)$; D $\left(\frac{r}{R_1} = \frac{R_1+3R_2}{4R_1} = 1.536\right)$; E $\left(1.714 = \frac{R_2}{R_1} \leq \frac{r}{R_1} \leq \frac{R_3}{R_1} = 1.969\right)$

Fig. 6 Fragment of the rotational mesh of finite elements.

the cable decomposition. The subject of investigations was the full model (Fig. 1), consisting of the three material zones. Because of the constant value of coefficient ε , the heat transfer takes place in radial direction only. It enabled approximation of the cable by the mesh of axially symmetrical finite elements. For approximation of the full model 80 rectangular elements were used in total of 409 nodes. The fragment of a mesh was shown in Fig. 6. Rotation of the rectangles around the axis of the model responds to a short section of the cable (a disc). The region of heat sources (i.e. a fragment of the core) was denoted by letter H. In the zone of strong decrease of the temperature (for example in the insulation) the mesh was refined (Fig. 6). Adiabatic boundary conditions were assumed on the bottom and top base of the model, because there is no heat transfer along the axis of a cable. A condition of the third kind [6] with the total coefficient of heat transfer was assumed on the side surface. It is a sum of the coefficients of convection and radiation.

The necessary data were taken from the set (36). The load of the same steady state current rating $I_{ll} = 789.8A$ was assumed. Computations were carried by means of the Nisa/Heat Transfer computer program [14], which was utilized for verification. The relative differences between heating curves were defined as follows

$$100\% \frac{[T_{FE}(r, t) - T_D(r, t)]}{T_{FE}(r, t)}, \quad (37)$$

where $T_D(r, t) = T_2(r, t)$, $T_{FE}(r, t)$ spatial-temporal distributions of the temperature achieved based on the Duhamel theorem and finite element method, respectively. The obtained results were presented in Fig. 7. The logarithmic scale was assumed on the abscissa axis (Fig. 7) for more convenient observation of (37) at the beginning of the transient state.

6 Final remarks

The following remarks result from the carried considerations:

- A) The obtained results have a good physical interpretation. It results from Fig. 5, that the velocity of rising and value of the presented heating curves are declined with the increase of the radial coordinate. Physical reason of

Fig. 7 Relative differences (37) of the heating curves determined by the method of finite elements (FE) and Duhamel's one (D) at the selected points of a cable: A $\left(0 \leq \frac{r}{R_1} \leq 1\right)$; B $\left(\frac{r}{R_1} = \frac{3R_1+R_2}{4R_1} = 1.179\right)$; C $\left(\frac{r}{R_1} = \frac{R_1+R_2}{2R_1} = 1.357\right)$; D $\left(\frac{r}{R_1} = \frac{R_1+3R_2}{4R_1} = 1.536\right)$; E $\left(1.714 = \frac{R_2}{R_1} \leq \frac{r}{R_1} \leq \frac{R_3}{R_1} = 1.969\right)$

the above phenomenon is a growing distance of the point of observation from the heat source (a core). Simultaneously reduction in a distance from the cooling surface R_3/R_1 takes place. The fundamental solution (31) shows similar properties (Fig. 3). However, because of zero temperature of the circle $r = R_2$ (Fig. 2a), respective curves decay significantly faster than in Fig. 5. Because boundary conditions of the solution (29) are inverse than (31) (Fig. 2b), direction of changes of the characteristic (29) is opposite (Fig. 4).

- B) Relative differences (37) of the heating curves computed by the method of finite elements (FE) and Duhamel's (D) are the greatest in the closed surroundings of the outer coating (Fig. 7, curve E). As it is seen in Fig. 7, deviations depend on position of the point, but they do not exceed 7.2%. In comparison to the paper [3], the deviations (37) slightly increased. It is caused by the approximation (1), (2). The considered differences come from the finite element method, as well, which brings largest discrepancies on the boundaries of material zones [15]. The thermal conductance of the core and coating in the analysed model is hundreds times greater than in insulation. It emphasizes the above discussed deviations. Selection of the time constants τ_1 , τ_3 has influence on accuracy of the obtained results, as well. Under- or over-estimation of time constants decreases the accuracy of the presented method.
- C) It is worth to mention, as well, about the time of computations and expenditure required by the both methods (FE and D). In case of the finite element method the time of computations was ca 160 s for 409 nodes with integration step of 1s. In addition the time needed for preparation of the model in the programme NISA [14] should be considered, as well. On the other hand the

time of computations in the Duhamel method was 15 s for five points (A–E) including drawing of the characteristics. The knowledge of heating curves at a few characteristic points is usually sufficient. For this reason the developed method (D) seems to be attractive. The FE method computes the field at all nodes of the mesh, which is often redundant.

The other serious advantage of the Duhamel method is the analytic form of final results. It simplifies discussion on the influence of particular parameters and physical interpretation of the solution.

It is evident, that the developed method D is connected with an extra time needed for derivation of the formulas and their programming in the Mathematica environment [12].

- D) In comparison to Gołębowski and Zaręba [3], the presented method significantly simplifies the transient thermal field analysis in a polymeric cable. Because it was proved, that it is sufficient to solve the equation of heat conduction (6) in one material zone only instead of three ones. Moreover, the relevant sequence of eigenfunctions is orthogonal (25). In the consequences of that coefficients of the field function are expressed in the form of Eqs. (28), (34) (in [3] they were determined in numerical way). Only one series of eigenvalues (22) is determined instead of three. This way the complex algorithm for their investigation is redundant [3]. Then the presented simplified method has many advantages.

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