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Planar millimetre wave structures

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Abstract Periodic structures in the millimetre wave range, with high precision requirements such as those needed for travelling wave tubes or for particle acceleration, must be planar structures for fabrication reasons. This paper treats single and double periodic devices semi-analytically with the mode matching technique. The method is explained for a side-coupled structure where Floquet's theorem was generalized for point-symmetric arrangements. With only slight modifications, the results can be adjusted to the case of on-axis coupled double-periodic structures. A special case is the standard single-periodic structure. Results show dispersion diagrams, field plots and beam coupling impedances.

Keywords Periodic structures · Planar mm-wave structures · Particle acceleration · Mode matching

1 Introduction

Recently, millimetre-wave travelling and standing wave structures have been proposed which are suited for travelling wave tubes or for electron acceleration. Because of the high mechanical precision required, the

only technologies fulfilling these requirements are wire electrodischarge machining (WEDM) and deep X-ray lithography (LIGA process [1]). In both cases, the structures must be planar. They consist of two opposing halves with a gap in-between. Each half supports a series of periodically arranged cavities. The first structure which was proposed was a single-periodic, travelling wave structure operating in a mode with a $2\pi/3$ phase advance per cell at 120 GHz [2]. Figure 1 shows the top view on the lower half of the structure. The upper half is identical. Since it is not straightforward to make planar, travelling wave structures with constant power dissipation, so-called constant gradient structures, standing wave structures were also investigated. But the problem with standing wave structures is that they have a high shunt impedance only for modes with zero or π phase advance per cell where mode separation and group velocity are small. Therefore, for a large number of accelerating cells, they are normally designed as double periodic, confluent structures. In confluent structures, the first two pass bands couple at the band edge, e.g. at the π phase advance, in such a way that group velocity and mode separation are large.

Initially, a so-called side-coupled double periodic structure (Fig. 2) was proposed [3]. Although the structure showed the right behaviour in principal, the results were not very satisfactory. The reason is that cell-to-cell coupling takes place not only between the main and side cells but also between the main cells. As a result, the dispersion diagram is strongly distorted. A better arrangement of the coupling cells is on-axis between the main cells (Fig. 3, [4]). In that case, the dispersion diagram is nearly symmetric, and the group velocity has a maximum at the confluence point. All these structures have been calculated numerically. Here, we shall develop a mode matching technique which allows us to make a semi-analytical treatment. As an example, we choose the side-coupled structure (Fig. 2) which has the most general geometry and from which the other geometries can be derived.

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Electr Eng (2004) 86:147–155 The equations throughout the printed contribution in the issue vol. 86, no. 3, pp. 147–155 were reproduced with many errors. The version published online 29 October 2003 is free of errors. For this reason, the complete contribution is being published as an Erratum.

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Fig. 1 Lower half of a symmetrical, single-periodic, planar travelling wave structure

Fig. 2 Lower half of a side-coupled planar standing wave structure

Fig. 3 Lower half of an on-axis coupled planar standing wave structure

2 Floquet's theorem for double-periodic structures

Planar structures are, by nature, fully three-dimensional and require a two-dimensional Fourier expansion of the fields. It is therefore obvious to take advantage of all existing symmetries and to reduce the computing efforts in that way.

Let us consider a two-dimensional double-periodic geometry, as given in Fig. 4. The figure shows schematically the electric and magnetic fields for the two possible polarizations and the case of a symmetric field configuration.

Fig. 4 A double-periodic structure with **a** perpendicular and **b** parallel polarization

From this and the antisymmetric case, we derive a generalized Floquet theorem

$$\begin{aligned} \mathbf{e}_y \times \mathbf{E}(-x, z + L_e) &= -\underline{\mathcal{D}}[\mathbf{e}_y \times \mathbf{E}(x, z)]e^{-j\beta_0 L_e} \\ \mathbf{e}_y \times \mathbf{H}(-x, z + L_e) &= +\underline{\mathcal{D}}[\mathbf{e}_y \times \mathbf{H}(x, z)]e^{-j\beta_0 L_e} \end{aligned} \quad (1)$$

$$\underline{\mathcal{D}} = \begin{pmatrix} -1 & 0 \\ 0 & +1 \end{pmatrix}$$

The effective period length L_e equals half of the total period length, $L_e = L_g/2$, and $\beta_0 L_g$ is the phase advance in a full geometrical period. The advantage of transformation (1) is immediately obvious when applying it to a geometry, as shown in Fig. 2. The mode matching procedure must be performed only over the effective period length, and the fields in the other cells are derived via Eq. (1).

3 Excitation by a charged particle

Applying the mode matching technique to the present boundary value problem normally yields a homogeneous system of linear equations. After truncating the system, the eigenvectors represent an approximate solution for the electromagnetic field problem and can be used to calculate the RF parameters of the structure, e.g. shunt impedance, Q-value and so on. However, this approach is numerically critical, and it is better to excite the field, i.e. to derive an inhomogeneous system of linear equations. As excitation, we take a particle of charge q travelling with constant speed $\mathbf{v} = \beta c_0 \mathbf{e}_z$ parallel to the z axis of the structure at position $(x, y) = (d_x, d_y)$.

The corresponding charge and current densities are

$$\begin{aligned} q_V(\mathbf{r}, t) &= q\delta(x - d_x)\delta(y - d_y)\delta(z - vt) \\ \mathbf{J}(\mathbf{r}, t) &= q_V(\mathbf{r}, t)v\mathbf{e}_z \end{aligned} \quad (2)$$

which after a Fourier transform in time

$$\delta(z - vt) \rightarrow \frac{1}{v} e^{-jkz/\beta}, \quad k = \frac{\omega}{c_0}, \quad \beta = \frac{v}{c_0}$$

and taking a current sheet in the plane $y = d_y$ yields

$$\mathbf{J}_F(\mathbf{r}, \omega) = q\delta(x - d_x)e^{-jkz/\beta}\mathbf{e}_z. \quad (3)$$

This current sheet interacts with the structure and excites synchronous Bloch waves at fixed frequencies. Although the energy per unit length of the Bloch waves is finite, its spectral width is infinitely small in loss-free structures, and therefore the amplitude is infinitely high. In order to deal with these singularities numerically, we introduce a complex wave number with a small imaginary part

$$k' = k(1 - j\delta), \quad 0 < \delta \ll 1. \quad (4)$$

Note, that the value of δ is insignificant as long as it is small. The energy deposited in the Bloch waves by the exciting current sheet is independent of δ , and RF parameters which do depend on losses can be corrected afterwards.

In the next step, we close the structure sideways in the planes $x = \pm L_x/2$ with electrically conducting walls. Thus, instead of a continuous spectrum, we obtain a set of discrete orthogonal functions for the x dependence. The influence of the walls can be kept small for sufficiently large L_x in the case where $d_y = 0$. The treatment of an open structure is given in [6] for the example of a single-periodic structure. It is also shown there how a vertical offset, $d_y \neq 0$, modifies the dispersion diagram.

Having derived the exciting current (3), we calculate the primary magnetic field, that is, the magnetic field which the current creates in a rectangular waveguide with dimensions L_x and $2a$ (Fig. 5). This waveguide corresponds to the planar structure with the cavities closed at the surface and the gap region closed at $x = \pm L_x/2$.

The current sheet creates purely transverse magnetic fields which can be calculated from a scalar function

$$\mathbf{B}^{(p)} = k'^2 \nabla \times (\mathbf{e}_z P), \quad \nabla^2 P + k'^2 P = 0. \quad (5)$$

Later, we need only the tangential magnetic field in the plane $y = a$ and obtain from (5)

$$\begin{aligned} \mathbf{e}_y \times \mathbf{H}^{(p)}(x, y = a) &= \frac{2q}{L_x} \sum_{m=1}^{\infty} \frac{\sin \eta_m (d_y + a)}{\sin 2\eta_m a} \\ &\times \sin \frac{m\pi(x + \frac{L_x}{2})}{L_x} \sin \frac{m\pi(d_x + \frac{L_x}{2})}{L_x} e^{-j\omega z/v} \mathbf{e}_z \end{aligned} \quad (6)$$

Fig. 5 Moving point charge in a rectangular waveguide

with

$$\eta_m = j \sqrt{k^2 \left(\frac{1}{\gamma^2 \beta^2} + 2j\delta \right) + \left(\frac{m\pi}{L_x} \right)^2}, \quad \gamma = \frac{1}{\sqrt{1 - \beta^2}}.$$

4 The secondary field of the cavities

After the primary field, Eq. (6), we must determine the secondary fields which are due to the cavities. Let us start with the geometry as shown in Fig. 6. Field expansions are used in cavity regions 1, 2 and in drift region 3. The fields in regions 1', 2', 2'' are calculated using Eq. (1). It is well known that a source-free field can be decomposed into TE and TM parts with respect to a Cartesian coordinate. We choose the y axis, which is perpendicular to the matching plane $y = a$.

Making use of the abbreviation

$$\sum_{m,n,TX} = \sum_{m=0}^{\infty} \sum_{\substack{n=0 \\ m+n \neq 0}}^{\infty} \text{TE}_{y-} + \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} \text{TM}_{y-} \quad (7)$$

we can write the field components which are tangential to the matching plane as

$$\begin{aligned} \mathbf{E}_t^{(1,2)} &= Z_0 \sum_{m,n,TX} B_{mn}^{(1,2)} \mathbf{G}_{mn}^{(1,2)}(\xi, \eta) W_{mn}^{(1,2)}(y) \\ \mathbf{e}_y \times \mathbf{H}^{(1,2)} &= \sum_{m,n,TX} B_{mn}^{(1,2)} \mathbf{G}_{mn}^{(1,2)}(\xi, \eta) W_{mn}^{(1,2)'}(y) Y_{mn}^{(1,2)} \\ \mathbf{E}_t^{(3)} &= Z_0 \sum_{m,n,TX} A_{mn} \mathbf{F}_{mn}(x, z) V_{mn}(y) \\ \mathbf{e}_y \times \mathbf{H}^{(3)} &= \sum_{m,n,TX} A_{mn} \mathbf{F}_{mn}(x, z) V'_{mn}(y) Y_{mn}^{(3)} \end{aligned} \quad (8)$$

Fig. 6a, b The side-coupled structure: a top view; b cross section

with

$$\mathbf{E}_t = -\mathbf{e}_y \times (\mathbf{e}_y \times \mathbf{E}).$$

Functions $\mathbf{G}_{mn}^{(1,2)}(\xi, \eta)W_{mn}^{(1,2)}(y)$ represent the standing wave solutions, while functions $\mathbf{F}_{mn}(x, z)V_{mn}(y)$ describe the waves travelling in the z direction with the phase constant for the n -th space-harmonic of

$$\beta_n = \beta_0 + \frac{2\pi n}{g_1 + t_1}, \quad n = 0, \pm 1, \pm 2, \dots \quad (9)$$

The functions are described in detail in the Appendix. ξ and η are local coordinates in the respective cells (see Fig. 6) and $Y_{mn}^{(i)}$ is the wave admittance normalized with respect to $Z_0 = \sqrt{\mu_0/\epsilon_0}$ for waves propagating in the y direction. The expansion coefficients A_{mn} and $B_{mn}^{(1,2)}$ are unknown.

5 Field matching

In the plane $y=a$, the following boundary and continuity conditions must be fulfilled

$$\mathbf{E}_t^{(3)} = \begin{cases} \mathbf{E}_t^{(1)}, & -\frac{w}{2} \leq x \leq +\frac{w}{2} \\ & \frac{t}{2} \leq z \leq \frac{t}{2} + g \\ \mathbf{E}_t^{(1')}, & -\frac{w}{2} \leq x \leq +\frac{w}{2} \\ & -\frac{t}{2} - g \leq z \leq -\frac{t}{2} \\ \mathbf{E}_t^{(2)}, & \frac{w}{2} + s \leq x \leq \frac{w}{2} + w_1 + s \\ & -g_1 \leq z \leq g_1 \\ \mathbf{E}_t^{(2')}, & -\frac{w}{2} - w_1 - s \leq x \leq -\frac{w}{2} - s \\ & \frac{t}{2} \leq z \leq \frac{t}{2} + g_1 \\ \mathbf{E}_t^{(2'')}, & -\frac{w}{2} - w_1 - s \leq x \leq -\frac{w}{2} - s \\ & -\frac{t}{2} - g_1 \leq z \leq -\frac{t}{2} \\ 0, & \text{elsewhere} \end{cases} \quad (10)$$

$$\mathbf{e}_y \times (\mathbf{H}^{(3)} + \mathbf{H}^{(p)}) = \begin{cases} \mathbf{e}_y \times \mathbf{H}^{(1)}, & -\frac{w}{2} \leq x \leq +\frac{w}{2} \\ & \frac{t}{2} \leq z \leq \frac{t}{2} + g \\ \mathbf{e}_y \times \mathbf{H}^{(1')}, & -\frac{w}{2} \leq x \leq +\frac{w}{2} \\ & -\frac{t}{2} - g \leq z \leq -\frac{t}{2} \\ \mathbf{e}_y \times \mathbf{H}^{(2)}, & \frac{w}{2} + s \leq x \leq \frac{w}{2} + w_1 + s \\ & -g_1 \leq z \leq g_1 \\ \mathbf{e}_y \times \mathbf{H}^{(2')}, & -\frac{w}{2} - w_1 - s \leq x \leq -\frac{w}{2} - s \\ & \frac{t}{2} \leq z \leq \frac{t}{2} + g_1 \\ \mathbf{e}_y \times \mathbf{H}^{(2'')}, & -\frac{w}{2} - w_1 - s \leq x \leq -\frac{w}{2} - s \\ & -\frac{t}{2} - g_1 \leq z \leq -\frac{t}{2} \end{cases} \quad (11)$$

After substituting expansions (8) into (10) and (11) and taking (1) into account, we use the orthogonal property of the functions $\mathbf{G}_{mn}(\xi, \eta)$ and $\mathbf{F}_{mn}(x, z)$ and obtain the following system of linear equations

$$\begin{aligned} S_{pq}^{(i)} &= Y_{pq}^{(i)} J_{pq}^{(i)} B_{pq}^{(i)} - \sum_{m,n,TX} M_{pq/mn}^{(i)*} V'_{mn}(a) Y_{mn}^{(3)} A_{mn}, \quad i=1,2 \\ A_{pq} I_{pq} &= \sum_{m,n,TX} \{(-1)^{m+q} - 1\} M_{mn/pq}^{(1)} W_{mn}^{(1)}(a) B_{mn}^{(1)} \\ &\quad + \sum_{m,n,TX} \{(-1)^{p+q} - 1\} M_{mn/pq}^{(2)} W_{mn}^{(2)}(a) B_{mn}^{(2)}. \end{aligned} \quad (12)$$

The terms $J_{pq}^{(i)}$, I_{pq} , which follow from the two-dimensional Fourier transform and the coupling integrals $M_{mn/pq}^{(i)}$, $S_{pq}^{(i)}$ are defined by

$$\begin{aligned} J_{pq}^{(1,2)} &= \int_0^{g,g_1} \int_0^{w,w_1} |\mathbf{G}_{pq}^{(1,2)}(\xi, \eta)|^2 d\xi d\eta \\ I_{pq} &= \int_0^{g_1+t_1} \int_0^{L_x} \mathbf{F}_{pq}(x, z) \mathbf{F}_{pq}^*(x, z) dx dz \\ M_{mn/pq}^{(1,2)} &= \int_0^{g,g_1} \int_0^{w,w_1} \mathbf{G}_{mn}^{(1,2)}(\xi, \eta) \mathbf{F}_{pq}^*(x, z) d\xi d\eta \\ S_{pq}^{(1,2)} &= \int_0^{g,g_1} \int_0^{w,w_1} \mathbf{G}_{pq}^{(1,2)}(\xi, \eta) [\mathbf{e}_y \times \mathbf{H}^{(p)}(x, y=a)] d\xi d\eta \end{aligned} \quad (13)$$

and the results are given in the Appendix. The terms $\{(-1)^{m+q}-1\}$ and $\{(-1)^{p+q}-1\}$ in Eq. (12) were generated by applying the generalized Floquet theorem (1) and must be dropped in the case when the coupling cells are arranged on-axis (Fig. 3).

6 Coupling impedance

The beam coupling impedance per unit length of a periodic structure with period L is defined as

$$Z'_{\parallel}(\omega) = -\frac{1}{qL} \int_0^L E_{\parallel}(\omega) e^{jkz/\beta} dz, \quad \mathbf{Z}'_{\perp} = \frac{\beta}{k} \nabla Z'_{\parallel} \quad (14)$$

where the subscripts refer to the impedances parallel and perpendicular, respectively, to the trajectory of the exciting charge q . As mentioned in Sect. 3, the real part of the impedance vanishes in loss-free structures, except at frequencies where the phase velocity of a Bloch wave matches the particle velocity v . Then, the impedance becomes infinite. In the present case, however, since we have introduced small losses, the resonances have finite width and the magnitude of the impedance becomes approximately

$$|Z(f)| = \frac{1}{2} \frac{R_c}{Q} \frac{1}{\sqrt{V^2 + Q^{-2}}}, \quad V = \frac{f}{f_r} - \frac{f_r}{f}. \quad (15)$$

Relation (15) is of particular value because it allows the resonance frequency f_r and the R/Q value at resonance to be determined. We sample the impedance at three different frequency points

$$\zeta_i = |Z(f_i)|^2, \quad V_i = \frac{f_i}{f_r} - \frac{f_r}{f_i}$$

and obtain from (15) the resonance frequency

$$f_r^4 = -\frac{f_{M-1}^2 - f_M^2 + \alpha(f_{M+1}^2 - f_{M-1}^2)}{f_{M-1}^{-2} - f_M^{-2} + \alpha(f_{M+1}^{-2} - f_{M-1}^{-2})} \quad (16)$$

$$\alpha = \frac{1 - \zeta_{M-1}/\zeta_M}{1 - \zeta_{M-1}/\zeta_{M+1}}$$

and the R/Q

$$\frac{R_c}{Q} = 2\sqrt{\frac{V_M^2 - V_{M-1}^2}{\zeta_M^{-2} - \zeta_{M-1}^{-2}}}. \quad (17)$$

Here, the index M designates the frequency point with the highest value ζ_i . The coupling impedance can be written in terms of the field expansion coefficients by substituting (8) into (14)

$$\frac{Z'_{||}}{Z_0} = \sum_{m,0,TX} A_{m0} F_{z,m0}(x,0) V_{m0}(y). \quad (18)$$

As can be seen in (18) and from the Appendix, only the synchronous waves, $n=0$, couple to the beam.

7 Stored energy and energy flow

For both the calculation of the stored energy and the calculation of the energy flow, the y components of the fields are required. From Maxwell's equations, we get

$$\begin{aligned} -j\omega\epsilon E_y &= \text{div}(\mathbf{e}_y \times \mathbf{H}) \\ -j\omega\mu H_y &= \mathbf{e}_y \cdot \text{rot } \mathbf{E}_t \end{aligned} \quad (19)$$

and therefore

$$\begin{aligned} E_y^{(1,2)} &= jZ_0 \sum_{m,n,TM} B_{mn}^{(1,2)} \frac{W_{mn}^{(1,2)'}(y)}{K_{mn}^{(1,2)}} \text{div } \mathbf{G}_{mn}^{(1,2)}(\zeta, \eta) \\ E_y^{(3)} &= jZ_0 \sum_{m,n,TM} A_{mn} \frac{V'_{mn}(y)}{K_{mn}^{(3)}} \text{div } \mathbf{F}_{mn}(x, z) \\ H_y^{(1,2)} &= \frac{j}{k} \sum_{m,n,TE} B_{mn}^{(1,2)} W_{mn}^{(1,2)}(y) \mathbf{e}_y \cdot \text{rot } \mathbf{G}_{mn}^{(1,2)}(\zeta, \eta) \\ H_y^{(3)} &= \frac{j}{k} \sum_{m,n,TE} A_{mn} V_{mn}(y) \mathbf{e}_y \cdot \text{rot } \mathbf{F}_{mn}(x, z). \end{aligned} \quad (20)$$

Since all field components are now expanded in series, the time-averaged stored energy can be evaluated. We use the fact that in a periodic structure the time-averaged magnetic energy equals the time-averaged electric energy and obtain

$$\bar{W}_e = p \left\{ \bar{W}_e^{(1)} + \bar{W}_e^{(2)} \right\} + \bar{W}_e^{(3)}$$

$$\begin{aligned} \frac{2\bar{W}_e^{(1,2)}}{\epsilon_0} &= \int_0^w \int_0^b \int_0^a \mathbf{E}^{(1,2)} \cdot \mathbf{E}^{(1,2)*} dx dy dz \\ \frac{2\bar{W}_e^{(3)}}{\epsilon_0} &= \int_0^{L_x} \int_0^a \int_0^L \mathbf{E}^{(3)} \cdot \mathbf{E}^{(3)*} dx dy dz \end{aligned} \quad (21)$$

with

$$p = \begin{cases} 1, & \text{on - axis coupled} \\ 2, & \text{side - coupled} \end{cases}$$

and

$$L = \begin{cases} g + g_1 + t + t_1, & \text{on - axis coupled} \\ 2(g + t), & \text{side - coupled} \end{cases}$$

The integrals are given in Appendix 5.

The energy flow can be evaluated in an arbitrary cross section $z=\text{const}$. For convenience, we consider the on-axis coupled structure because, for a proper choice of the location of the cross section, we only need the fields in the drift space 3 (Fig. 6). Then the transported power is

$$\bar{P} = \text{Re} \int_0^{L_x} \int_0^a \left(E_x^{(3)} H_y^{(3)*} - H_x^{(3)*} E_y^{(3)} \right)_{z=0} dy dx \quad (22)$$

where the integral is given in Appendix 6.

8 Shunt impedance and group velocity

Having derived the stored energy and the energy flow, we can calculate the shunt impedance R_s and the group velocity v_g .

The shunt impedance per cell is defined as

$$R_s = \frac{\left| \int_0^L E_z e^{jkz/\beta} dz \right|^2}{\bar{P}_v} \quad (23)$$

with $\bar{P}_v$ the time-averaged dissipation. The R upon Q is given by

$$\frac{R_s}{Q} = \frac{Z_0 \left| \int_0^L E_z e^{jkz/\beta} dz \right|^2}{k \frac{2\bar{W}_e/\epsilon_0}{2\bar{W}_e/\epsilon_0}} \quad (24)$$

where the denominator was derived above. The numerator, that is the longitudinal voltage, follows from

$$\left| \int_0^L E_z e^{jkz/\beta} dz \right| = Z_0 L \left| \sum_{m,n,TX} (-1)^n \frac{\sin \alpha_n}{\alpha_n} A_{mn} F_{z,mn}(x,0) V_{mn}(y) \right| \quad (25)$$

with

$$\alpha_n = \frac{L}{2} \left(\frac{k}{\beta} - \beta_n \right).$$

Note that, for the synchronous field, $\beta_0 = k/\beta$, the sum reduces to the term with $n=0$.

Because of the relation

$$\frac{R_s}{Q} = 2 \frac{Z_{||}(\omega = \omega_r)}{Q} = \frac{R_c}{Q} \quad (26)$$

one can calculate the shunt impedance in three different ways:

1. One uses the approximate relation (17), which follows from sampling at three points around the resonance.

2. One searches the maximum of the coupling impedance divided by the Q -value

$$Q = \frac{\omega \bar{W}}{P_v} \approx \frac{1}{2\delta} \quad (27)$$

in case of dielectric losses only.

3. The shunt impedance is calculated as given in Eq. (24).

The comparison of the different results is an excellent check. In general it turned out that methods (2) and (3) give similar results. However, the calculation of $Z_{||}'(\omega = \omega_r)$ must be done thoroughly, especially in the case of small losses. Occasionally, small differences to method (1) occurred because the assumed frequency dependence in Eq. (15) differs from the actual dependence.

For the calculation of the group velocity, two different approaches are possible. Either we determine the slope of the dispersion diagram

$$\frac{v_g}{c_0} = 1.2 \frac{L}{[\text{mm}]} \frac{\Delta f [\text{GHz}]}{\Delta \phi / [\text{deg}]} \quad (28)$$

or we use the ratio of energy flow to stored energy per unit length

$$\frac{v_g}{c_0} = LZ_0 \frac{\bar{P}}{2\bar{W}_c / \epsilon_0}. \quad (29)$$

The results of both approaches agree very well. Finally, Table 1 shows the shunt impedance and the

Table 1 Comparison of the three structures

Structure	Single-periodic	Side-coupled	On-axis-coupled
$f/[\text{GHz}]$	122	94	91.392
$\phi/[\text{deg}]$	122	180	180
R_c/Q [kΩ/m]	144.7	78.6	64
R_s/Q [kΩ/m]	138.3	78.4	62.5
$2Z(\omega_r)/Q$ [kΩ/m]	138.3	78.4	62.5
v_g/c_0	4.4%	1.2%	10%

group velocities derived in the different ways for the three structures.

9 Modifications for the on-axis coupled structure

The above formulae refer to the side-coupled structure as shown in Fig. 2, for which the generalized Floquet theorem is valid. For the on-axis coupled structure, Fig. 3, we must modify the formulae slightly:

1. Factors $\{(-1)^{m+q}-1\}$ and $\{(-1)^{p+q}-1\}$ in Eq. (12) must be dropped,

2. Because of the change in location of the coupling cells, the coordinate transformation

$$\Delta \xi^{(2)} \rightarrow \frac{L_x - w_1}{2}, \quad \Delta \eta^{(2)} \rightarrow g + t_1 + \frac{t}{2}$$

must be performed in the coupling matrices in Appendix 4.

3. The period length is now

$$L = g + t + g_1 + t_1.$$

10 Numerical results

10.1 Single-periodic structure

Let us start with a single-periodic structure as shown in Fig. 1 and proposed in [2]. The structure was designed for a $2\pi/3$ mode at 120 GHz. We suppress the excitation of the structure, that is the term $S_{pq}^{(1,2)}$ in Eq. (12) vanishes, and obtain a homogeneous system of equations, the eigenvalues of which give the dispersion diagram Fig. 7. It corresponds well to the published data. The coupling impedance for the $2\pi/3$ mode is shown in Fig. 8. The impedance in Fig. 8 was calculated at seven points only and interpolated. Using the three-point formulae given in Sect. 6, we obtain a resonance frequency of 122 GHz and a R/Q value of 144.7 kΩ.

Fig. 7 Dispersion diagram for the single periodic structure proposed in [2]. The geometrical parameters refer to Fig. 6 and are given by $a=0.3$, $b=0.9$, $w=1.8$, $g=0.633$, $t=0.2$, $L_x=3.6$, all in mm, $\beta_0 L = 2\pi/3$

Fig. 8 Longitudinal coupling impedance for the structure in Fig. 7: **a** $\text{Im}\{k\}/|k| = 10^{-4}$; **b** $\text{Im}\{k\}/|k| = 10^{-5}$

Obviously the procedure is nearly independent of the position of the sampling points and of the assumed losses. In particular, since the R/Q does not change with the losses, the total energy lost in this mode, that is the integral over the real part of Z , is independent of the losses.

10.2 Side-coupled structure

As mentioned in Sect. 1, double-periodic structures are used in order to operate at a point of confluence where the group velocity and the mode separation are large. To find the point of confluence, we calculate the contour $w_1 = w_1(w)$ from the requirement of a vanishing system determinant

$$\text{determinant } \{X(f, w, w_1)\} = 0.$$

Here, w and w_1 are the widths of the main and side cells, respectively, and X is the system matrix of Eq. (12). Figure 9 shows such a contour. It can be seen that the

Fig. 9 Contour $w_1 = w_1(w)$ for vanishing real part (solid line) and imaginary part (dashed line) of the system determinant in case of a 94-GHz side-coupled structure operating in π mode. $a=0.375$, $b=1.145$, $g=1.345$, $t=s=0.25$, $L_x=10$, all in mm

Fig. 10 Dispersion diagram of a confluent side-coupled structure (dimensions as in Fig. 9 and $w = 2.1$, $w_1 = 2.253$, all in mm)

real part consists of two branches which approach each other at the point of confluence K. The distance depends on the losses (here $\text{Im}\{k\} = 0.001$). For vanishing losses, the two branches connect at K and determine the values of w and w_1 .

The corresponding dispersion diagram is given in Fig. 10. Clearly, although confluence happens, group velocity and mode spacing are not optimal. The reason is that coupling takes place from main-to-main cells and from main-to-coupling cells thus heavily distorting the dispersion diagram.

10.3 On-axis coupled structure

Similar to the side-coupled structure, the on-axis coupled structure, Fig. 3, should operate in a confluent π mode. The operational frequency is 91.392 GHz, a frequency which is used in the Stanford Linear Accelerator Center. The dispersion diagram and the geometrical dimensions of the structure are given in Fig. 11.

Since the structure is symmetrical with respect to the plane $x=0$, we may draw the lines

$$H_x = \text{const.}$$

which are shown in Fig. 12. If the structure were two-dimensional, i.e. independent of x , the lines would represent the displacement current. In the present three-dimensional structure, however, they are only an approximation of the displacement current of the fundamental mode at $f=91.392$ GHz. The figure shows the field pattern over one quarter of the RF period. Obviously, boundary and continuity conditions are well fulfilled. Also note the field energy, which is transported from the main cells into the coupling cells within a quarter of a period.

Fig. 11 Dispersion diagram of a confluent on-axis coupled structure with $a=0.525$, $b=1.27$, $w=2.237$, $w_1=3.494$, $g=0.98$, $g_1=0.2$, $t=t_1=0.23$, $L_x=4.4$, all in mm

Fig. 12 Lines of constant H_x in the symmetry plane $x=0$ of the on-axis coupled structure. Parameter is time

11 Conclusions

In this paper, the mode matching analysis of single periodic planar millimetre wave structures, as elaborated in an early work [5], is generalized to double periodic devices.

In contrast to the usual structures with circular cross section, the necessary Fourier analysis must be performed in two dimensions, so we have incorporated symmetry conditions in Floquet's theorem, which allow for reducing the numerical effort by a factor of four in case of a point-symmetric arrangement.

The relevant HF parameters are evaluated for three different devices, i.e. for a single periodic, a side-coupled and an on-axis coupled structure. Furthermore, it is shown that the shunt impedance can be calculated in a very effective manner via the beam coupling impedance. For this purpose, small dielectric losses were introduced such that the resonance peaks of the coupling impedance have finite width. Then it is sufficient to determine only three values in the vicinity of a resonance to obtain the resonance frequency as well as the shunt impedance R_s divided by the Q value.

By means of the developed mode matching computer code, we were able to design the double periodic structure in such a way that confluence of two pass-bands occurs at the pi-mode for a desired frequency. The parameters involved in the process are only the width of the main and coupling cells. As a result, the on-axis coupled structure shows better behaviour because of the large group velocity (about 10% of the velocity of light) at confluence.

Appendix 1. Orthogonal eigenfunctions

Region 1

$$\begin{aligned} \mathbf{G}_{mn}^{(1)} &= \mathbf{e}_x G_{x,mn}^{(1)} + \mathbf{e}_z G_{z,mn}^{(1)} \\ G_{x,mn}^{(1)} &= \sin \frac{n\pi y}{g} \cos \frac{m\pi z}{w} \begin{cases} \frac{n\pi}{g}, & TE_y \\ \frac{m\pi}{w}, & TM_y \end{cases} \\ G_{z,mn}^{(1)} &= \cos \frac{n\pi y}{g} \sin \frac{m\pi z}{w} \begin{cases} -\frac{m\pi}{w}, & TE_y \\ \frac{n\pi}{g}, & TM_y \end{cases} \end{aligned}$$

Region 2

As in region 1 with $w, g \rightarrow w_1, g_1$

Region 3

$$\begin{aligned} \mathbf{F}_{mn} &= \mathbf{e}_x F_{x,mn} + \mathbf{e}_z F_{z,mn} \\ F_{x,mn} &= e^{-j\beta_n z} \cos \frac{m\pi(x+\frac{L_x}{2})}{L_x} \begin{cases} j\beta_n, & TE_y \\ \frac{m\pi}{L_x}, & TM_y \end{cases} \\ F_{z,mn} &= e^{-j\beta_n z} \sin \frac{m\pi(x+\frac{L_x}{2})}{L_x} \begin{cases} -\frac{m\pi}{L_x}, & TE_y \\ -j\beta_n, & TM_y \end{cases} \end{aligned}$$

Appendix 2. Non-orthogonal functions of the field expansion

$$W_{mn}^{(i)}(y) = -\frac{W_{mn}^{(i)+}(y) - W_{mn}^{(i)-}(y)}{W_{mn}^{(i)+}(a) + W_{mn}^{(i)-}(a)}, \quad W_{mn}^{(i)'} = \frac{1}{jK_{mn}^{(i)}} \frac{\partial W_{mn}^{(i)}}{\partial y}$$

$$V_{mn}(y) = -\frac{V_{mn}^+(y) + V_{mn}^-(y)}{V_{mn}^+(a) + V_{mn}^-(a)}, \quad V'_{mn} = \frac{1}{jK_{mn}^{(3)}} \frac{\partial V_{mn}}{\partial y}$$

$$W_{mn}^{(i)\pm} = \exp \{ \mp j K_{mn}^{(i)} (y - b) \}$$

$$V_{mn}^{\pm} = \exp \{ \mp j K_{mn}^{(3)} y \}$$

$$K_{mn}^{(1)} = \sqrt{k^2 - \left(\frac{n\pi}{g}\right)^2 - \left(\frac{m\pi}{w}\right)^2}$$

$$K_{mn}^{(2)} = \sqrt{k^2 - \left(\frac{n\pi}{g_1}\right)^2 - \left(\frac{m\pi}{w_1}\right)^2}$$

$$K_{mn}^{(3)} = \sqrt{k^2 - \beta_n^2 - \left(\frac{m\pi}{L_x}\right)^2}$$

$$Y_{mn}^{(i)} = \begin{cases} K_{mn}^{(i)}/k, & TE_y \\ k/K_{mn}^{(i)}, & TM_y \end{cases} \quad i = 1, 2, 3$$

Appendix 3. Norms of the eigenfunctions

$$J_{mn}^{(1)} = \frac{gw}{4} (1 + \delta_0^m) (1 + \delta_0^n) \left\{ \left(\frac{m\pi}{w}\right)^2 + \left(\frac{n\pi}{g}\right)^2 \right\}$$

$$J_{mn}^{(2)} = \frac{g_1 w_1}{4} (1 + \delta_0^m) (1 + \delta_0^n) \left\{ \left(\frac{m\pi}{w_1}\right)^2 + \left(\frac{n\pi}{g_1}\right)^2 \right\}$$

$$I_{mn} = \frac{L_x L_g}{2} (1 + \delta_0^m) \left\{ \beta_n^2 + \left(\frac{m\pi}{L_x}\right)^2 \right\}$$

Appendix 4. Coupling integrals

Interface region 1/3

$$M_{mn/pq}^{(1)} = \sigma_{mp}^{(1)} \tau_{nq}^{(1)} \begin{cases} j\beta_q \frac{p\pi}{L_x} \left[\left(\frac{m\pi}{w}\right)^2 + \left(\frac{n\pi}{g}\right)^2 \right] & , TE/TE \\ \left(\frac{m\pi}{w}\right)^2 \beta_q^2 - \left(\frac{p\pi}{L_x}\right)^2 \left(\frac{n\pi}{g}\right)^2 & , TE/TM \\ 0 & , TM/TE \\ -\frac{m\pi}{w} \frac{n\pi}{g} \left[\left(\frac{p\pi}{L_x}\right)^2 + \beta_q^2 \right] & , TM/TM \end{cases}$$

$$\sigma_{mp}^{(1)} = \frac{(-1)^{m+1} \sin \frac{p\pi(w+\Delta\xi^{(1)})}{L_x} + \sin \frac{p\pi\Delta\xi^{(1)}}{L_x}}{\left(\frac{m\pi}{w}\right)^2 - \left(\frac{p\pi}{L_x}\right)^2}$$

$$\tau_{nq}^{(1)} = \frac{(-1)^n \exp(j\beta_q g - 1)}{\left(\frac{n\pi}{g}\right)^2 - \beta_q^2} \exp(j\beta_q \Delta\eta^{(1)})$$

$$\Delta\xi^{(1)} = \frac{L_x - w}{2}, \quad \Delta\eta^{(1)} = \frac{t}{2}$$

Interface region 2/3

As for transition 1/3 with $w, g \rightarrow w_1, g_1$ and

$$\Delta\xi^{(2)} = \frac{L_x + w}{2} + s, \quad \Delta\eta^{(2)} = -\frac{g_1}{2}$$

Appendix 5. Stored field energy

$$\begin{aligned} \frac{2\bar{W}_e^{(1,2)}}{\varepsilon_0} &= -Z_0^2 \sum_{m,n,TM} J_{mn}^{(1,2)} \left| \frac{B_{mn}^{(1,2)}}{K_{mn}^{(1,2)}} \right| P_{mn}^{(1,2)}(a-b) \\ &\quad \times \left\{ \left(\frac{m\pi}{w, w_1}\right)^2 + \left(\frac{n\pi}{g, g_1}\right)^2 \right\} \\ &\quad - Z_0^2 \sum_{m,n,TE} J_{mn}^{(1,2)} |B_{mn}^{(1,2)}|^2 Q_{mn}^{(1,2)}(a-b) \end{aligned}$$

$$\begin{aligned} \frac{2\bar{W}_e^{(3)}}{\varepsilon_0} &= +Z_0^2 \sum_{m,n,TM} I_{mn} \left| \frac{A_{mn}}{K_{mn}^{(3)}} \right| Q_{mn}^{(3)}(a) \left\{ \left(\frac{m\pi}{L_x}\right)^2 + \beta_n^2 \right\} \\ &\quad + Z_0^2 \sum_{m,n,TE} I_{mn} |A_{mn}|^2 P_{mn}^{(3)}(a) \end{aligned}$$

$$P_{mn}^{(i)}(y) = \frac{\text{Im}\{K_{mn}^{(i)} \tan(K_{mn}^{(i)} y)\}}{2\text{Re}\{K_{mn}^{(i)}\} \text{Im}\{K_{mn}^{(i)}\}}$$

$$Q_{mn}^{(i)}(y) = \frac{\text{Im}\{K_{mn}^{(i)} \tan(K_{mn}^{(i)*} y)\}}{2\text{Re}\{K_{mn}^{(i)}\} \text{Im}\{K_{mn}^{(i)}\}}, \quad i = 1, 2, 3$$

Appendix 6. Energy flow

$$\begin{aligned} &\int_0^{L_x} \int_0^a \mathbf{e}_z \cdot (\mathbf{E}^{(3)} \times \mathbf{H}^{(3)*})_{z=0} dy dx \\ &= \frac{Z_0 L_x}{jk} \sum_m \sum_n \sum_v \left\{ \left(\frac{m\pi}{L_x}\right)^2 + \beta_v^2 \right\} \\ &\quad \times \left\{ f_{mnv} A_{mv}^{TE*} \left[j\beta_n A_{mn}^{TE} + \frac{m\pi}{L_x} A_{mn}^{TM*} \right] \right. \\ &\quad \left. + f'_{mnv} A_{mv}^{TM} \left[\frac{j\beta_n k^2}{K_{mn}^{(3)*} K_{mv}^{(3)}} A_{mn}^{TM} - \frac{m\pi}{L_x} \frac{K_{mn}^{(3)*}}{K_{mv}^{(3)}} A_{mn}^{TE*} \right] \right\} \end{aligned}$$

$$f_{mnv} = \frac{K_{mn}^{(3)} \tan(K_{mn}^{(3)} a) - K_{mv}^{(3)*} \tan(K_{mv}^{(3)*} a)}{K_{mn}^{(3)2} - K_{mv}^{(3)*2}}$$

$$f'_{mnv} = \frac{K_{mv}^{(3)} \tan(K_{mn}^{(3)*} a) - K_{mn}^{(3)*} \tan(K_{mv}^{(3)} a)}{K_{mn}^{(3)*2} - K_{mv}^{(3)2}}$$

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