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EDITORIAL

Editorial

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Applied Topology is a loosely defined area where mathematicians interact with biologists, engineers, computer scientists, data analysts and others attracted by the belief that the flexibility of topological constructions can be a powerful force in applications. Some of the applications are purely theoretical, though many rely on efficient computer algorithms that apply to topological spaces modelling real-life problems. Such spaces are typically cellular, compact and can involve enormous amounts of data. There has been a huge growth of interest in Applied Topology in recent years. This special double volume of AAECC contains twelve contributions to the area on topics including topological data analysis, concurrency theory, algorithmic topology, and applications to combinatorics.

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1 Topological data analysis

Several of the contributions concern the problem of interpreting large data sets using topological invariants of associated cellular spaces. The paper by Vejdemo-Johansson et al. uses persistent cohomology to detect, parameterize and interpolate between periodic motion patterns in human walking and running gaits stored as motion capture sequences.The paper by Brendel et al. uses a discrete Morse theory algorithm for computing fundamental groups of cellular spaces to detect knots in the backbones of small proteins. The paper by Reina-Molina et al. uses bio-inspired models of membrane computing and discrete Morse theory to implement a framework for computing with digital images; the framework is used to design a parallel algorithm for computing homology groups of 3-dimensional binary digital images. The paper by Botnan and Spreemann uses homotopy collapses to derive an efficient algorithm for computing the persistent homology of a sequence of maps of simplicial complexes. The paper by Real et al. investigates the difficult problem of constructing a discrete vector field on a cubical space with the minimal number of critical cells, and a related and more tractable 'homological optimality' problem. The paper by Belchí and Murillo introduces and studies a notion of A_∞-persistence on the homology of a filtered topological space.

2 Concurrency theory

Concurrency theory studies systems in which several computations are executing simultaneously and potentially interacting with each other. Directed homotopies on directed cubical complexes is one notion used to model such systems. In this context the paper by Ottosen proves that the space of directed paths on the k-skeleton of the n-cube is homotopy equivalent to the nerve of a certain category of flags of finite sets. The paper by Misamore introduces a new method for simplifying finite directed cubical complexes. This leads to computations of morphism sets of path categories of such complexes in many cases of practical interest.

3 Computations in classical algebraic topology

Explicit computations of homotopy invariants of a CW-space X have been a central motivation in algebraic topology. The paper by Baues and Bleile describes a new method for computing the third homotopy group, $\pi_3 X$, as a module over $\pi_1 X$. The homotopy group $\pi_3 X$ is determined as an extension of $\pi_1 X$—modules derived from Whitehead's Certain Exact Sequence. The paper carries out explicit computations for pseudo-projective 3-spaces $X = S^1 \cup e^2 \cup e^3$ consisting of exactly one cell in each dimension ≤ 3.

4 Combinatorics

Three contributions concern topological applications to combinatorics. The paper by Álvarez et al. describes a method for constructing a basis for n-cocycles over a group

G, from which the whole set of n-dimensional n-cocyclic matrices over G can be straightforwardly calculated. Of special interest is the case $n = 2$ which is related to the calculation of cocyclic Hadamard matrices. The paper provides examples, for $n = 2, 3$ and includes examples of improper 3-dimensional 3-cocyclic Hadamard matrices. The paper by Costoya and Viruel shows how the vertex coloring problem for finite simplicial complexes can be translated into the algebraic homotopy problem of ellipticity for rational spaces. The paper follows ideas of Lechuga-Murillo for the classical vertex coloring of graphs. The paper by Deniz shows that a certain acyclic simplicial complex associated to a trisectionable tournament T is homotopy equivalent to a wedge of spheres and provides an upper bound on the (acyclic) chromatic number of T.