Computational algebraic geometry and its applications

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Forward from the editor

This issue of the Applicable Algebra in Engineering, Communication and Computing presents selected papers on work presented at, or related to, the first two meetings of the Michigan Computational Algebraic Geometry group, namely MCAG 2012, MCAG 2013. MCAG conferences are a yearly event held in one of the public universities in Michigan. Their aim is to bring together algebraic geometers from academia and industry and to foster further cooperation.

MCAG-group is a network of algebraic geometers in Michigan and surrounding areas in order to encourage collaboration, exploration of new applications, and stronger connections to industry. The group organizes yearly conferences in different Michigan institutions, training sessions for students, and encourages collaboration among different mathematics departments from Michigan. During the first two MCAG conferences, there were hundred of participants from all public research universities in Michigan. There were also participants from Ontario, Canada and many Midwest states. Among invited speakers were Igor Dolgachev (University of Michigan), William Fulton (University of Michigan), Gene Freudenburg (Western Michigan University), Rajesh Kulkarni (Michigan State University), Harm Derksen (University of Michigan), Charles Wampler (General Motors), Juan Migliore (University of Notre Dame), Annegret Paul (Western Michigan University), Gregory Pearlstein (Michigan State University), Tony Shaska (Oakland University), David Speyer (University of Michigan), Jan Verschelde (University of Illinois—Chicago), Fabrizio Zanella (Michigan Tech.), Dan Erman (University of Michigan), Cynthia Vinzant (University of Michi-

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gan), Qëndrim Gashi (Univ. of Prishtina), Susan Cooper (Central Michigan University), K. Lee (Wayne State University), Li Li (Oakland University), Zach Teitler (Boise State University), et al. The MCAG 2012 was held at Rochester (Oakland University) and MCAG 2013 was held in Kalamazoo (Western Michigan University). MCAG 2014 will be held in Ann Arbor (University of Michigan) during the month of May 2014.

While the focus of MCAG conferences is on computational aspects of algebraic geometry, we encourage participants and talks from all areas and applications of algebraic geometry. There is a particular interest on applications of algebra and algebraic geometry in coding theory and cryptography. The articles selected for this special issue represent this direction of the group. Here is a summary of the topics covered in this issue.

Frank-Olaf Schreyer, Martin Weimann, and Josef Schicho study computational aspects of gonial maps and radical parametrization of curves. They develop an algorithm that, given a projective curve C , computes a *gonal map*, that is, a finite morphism from C to $\mathbb{P}^1$ of minimal degree. Our method is based on the computation of scrollar syzygies of canonical curves. We develop an improved version of our algorithm for curves with a unique gonal map and we discuss a characterization of such curves in terms of Betti numbers. Finally, we derive an efficient algorithm for radical parametrization of curves of gonality ≤ 4 .

Eduardo Ballico and Alberto Ravagnani study the minimum distance and the minimum weight of Goppa codes from a quotient of the Hermitian curve. The geometry of many evaluation codes on the plane quotients Y_m of the Hermitian curve defined over $\mathbb{F}_{q^2}$ by the equations $y^q + y = x^m$, m being a proper divisor of $q + 1$ are considered. They either find out their dual minimum distance, or give a lower bound for it. In many cases they completely describe the minimum-weight codewords and give their number. Finally, the result is applied to certain classical Goppa codes on Y_m , such as one-point and two-point codes.

N. Aydin, S. Karadeniz, B. Yildiz, consider quasi-cyclic codes over the ring $\mathbb{R}$, a finite non-chain ring that has been recently studied in coding theory. The Gray images of these codes are shown to be binary quasi-cyclic codes. Using this method we have obtained seventeen new binary quasi-cyclic codes that are new additions to the database of binary quasi-cyclic codes. Moreover, is also obtained a number of binary quasi-cyclic codes with the same parameters as best known binary linear codes that otherwise have more complicated constructions.

Aysegul Bayram and Irfan Siap study the structure of linear codes over the non chain ring $Z_3[v]/(v^3 - v)$. In order to study the codes, they first study the structure of this ring via a distance preserving Gray map which also induces a relation between codes over this ring and ternary codes. Further, the algebraic structure of cyclic and dual codes is also studied. A MacWilliams type Identity between the Gray weight enumerators of the original code and its dual is established. Also, a BCH-type bound and an example that attains this bound is presented.

Fred Thompson and Tony Shaska study bielliptic curves of genus 3 defined over an algebraically closed field k and the intersection of the moduli space $\mathcal{M}_3^b$ of such curves with the hyperelliptic moduli $\mathcal{H}_3$. Such intersection $\mathcal{S}$ is an irreducible, 3-dimensional, rational algebraic variety. They determine the equation of this space in

terms of the $GL(2, k)$ -invariants of binary octavics and find a birational parametrization of $\mathcal{S}$. All possible subloci of curves for all possible automorphism group G are also computed. Moreover, for every rational moduli point $\mathfrak{p} \in \mathcal{S}$, such that $|\text{Aut}(\mathfrak{p})| > 4$, a rational model of the corresponding curve over its field of moduli is explicitly given in terms of the $GL(2, k)$ -invariants.

On putting together this special issue I had the help and support of many people. My thanks go to the Office of the Provost and the Office of Research Administration at Oakland University for providing a grant for MCAG 2012, the Meadow Brook Hall Foundation for allowing us to use their wonderful facilities, the National Security Agency (NSA) for the conference grant we received in organizing this conference, and the Department of Mathematics and Statistics at Oakland University for handling all the administrative tasks of the conference.

Special thanks go to the organizers of the MCAG 2012 and MCAG 2013 for an excellent job in organizing the conferences and to all the speakers and participants who made both conferences a success.

We hope the reader enjoys this collection of papers and finds the MCAG conferences an interesting venue of cooperation.