Symposium on revealed preference analysis

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Almost invariably, economic models postulate that agents behave according to some type of maximizing behavior. This is true even of models in behavioral economics, though agents in those settings may be unsophisticated in some way or have preferences that depart from classical assumptions. Revealed preference (RP) analysis investigates the observable implications of economic models and the extent to which the objects of a model, for example, agents' preferences, can be inferred from data. While RP analysis in this broad sense applies, of course, to a very large part of empirical investigation in economics, that literature which is typically understood by its practitioners as revealed preference analysis has more specific features: In general, the aim of the analysis is not merely to identify and test *some* empirical implications of a model, but to identify *all* the observable implications of a model. These implications take the form of restrictions on the data that could, at least in principle, be checked. Since the models being considered are usually sufficiently general that its objects are restricted by qualitative, rather than functional-form or parametric conditions, the same is true of the restrictions on the data. This is an ambitious approach to empirical investigation, and it is probably not practicable to adopt it in all circumstances, but for models that play a central role in economic analysis, it is worthwhile delineating *exactly* the observable content of the model.

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There is no more canonical model in economics than that of a consumer maximizing utility subject to a budget constraint, so it is not surprising that the early RP literature should have developed around the study of that model. The starting point of the analysis is the Marshallian demand function, which specifies the agent's demand bundle at different prices and incomes. The first question to ask is what conditions such a function must satisfy if it arises from a utility-maximizing consumer. After that, it is natural to pose the question distinctive to RP analysis: Are those conditions on the demand function also *sufficient*, in the sense that for any demand function satisfying them, one could find a utility function that generates that demand function? This is known as the *integrability problem* and is the focus of a large and significant literature (see [Mas-Colell et al. 1995](#) and its references).

The inspiration for much of the contributions to this symposium comes from a different and more recent strand of the RP literature. This literature, whose seminal contributions include [Afriat \(1967\)](#), [Richter \(1966\)](#), [Varian \(1982\)](#), departs from the earlier work in that it no longer requires that the observer has access to a complete set of observations. Instead, it assumes a more realistic setting in which observations are only partial and then seeks to identify the conditions on those partial observations that are necessary and sufficient for consistency with the model. The result which came to be known as Afriat's Theorem assumes that there is a *finite* set of observations, with each observation consisting of the prices faced by an agent for different goods and the bundle he chooses to buy at those prices. Different observations arise from different bundles bought at different price vectors. Afriat's Theorem says that if the agent is maximizing a locally non-satiated preference, the data set will obey a property called the generalized axiom of revealed preference (GARP, for short); the deeper half of the theorem says that any data set that obeys GARP can be rationalized as demand observations from an agent with a strongly monotone, continuous, and concave utility function (see [Fostel et al. 2004](#) for a recent proof). An important feature of Afriat's Theorem is that GARP, which is a no cycling condition on the revealed preference patterns found in the data, can be easily checked. Indeed, it is equivalent to the existence of a solution to a linear program; furthermore, the solution to this program yields the parameters that enter into an explicit formula for the rationalizing utility function. So the theorem provides an easily implementable test for the utility maximization hypothesis and, partly because of this, it has given rise to a significant empirical literature.

The starting point of Afriat's Theorem—a data set consisting of demand bundles and prices—seems so natural that one is hard pressed to think of an alternative. In other models, however, the starting point is not so obvious, or rather there are various plausible ways in which data could be generated. This is always the first and, sometimes, the most important issue that a piece of RP analysis must resolve and it will have an impact on whether, and how, a model could be tested through a set of RP conditions. For example, in developing RP tests for an oligopoly model, one could imagine data being generated by changes to the demand functions, with firms' cost functions staying fixed in the observation period (as in [Carvajal et al. 2013](#)), or the opposite, where demand functions are stable, while cost functions change and generate the data. What is reasonable depends on the context and the data that are, or are likely to be, available when implementing the test.

Afriat's theorem has been generalized in different ways. Given the formal similarities between consumer and producer theory, it is not surprising that one could carry out RP analyses of the latter that follow from, or are related to, Afriat's Theorem (see, for example, [Varian 1984](#)). It is natural to ask how one could characterize consumer data sets that arise from utility maximization where the utility function is required to have stronger properties like homotheticity, weak separability, or additivity (see, for example, [Varian 1983](#); [Green and Srivastava 1986](#); [Quah 2012](#)). Utility maximization may not be such a good model of behavior if agents live in households and share consumption with other household members. An RP analysis of the efficient household model studied by Chiappori and his co-authors (see [Chiappori 1988](#)), which assumes that the household always chooses a Pareto-optimal outcome, was undertaken by [Cherchye et al. \(2007\)](#). RP analysis can also be performed on equilibrium models where there are multiple agents interacting in an economy or game. [Brown and Matzkin \(1996\)](#) exploit Afriat's Theorem to develop a test for the Walrasian model; their framework has in turn been extended in various ways to account for externalities and other phenomena (see, for example, [Carvajal 2010](#)).

An important issue which arises in some of the RP tests developed for these models is that they may not be computationally straightforward. This is true, for example, of Varian's test of weak separability as well as Brown and Matzkin's test of the Walrasian model. In both cases, one is required to check that there is a solution to a set of quadratic inequalities. Such problems are solvable in that there is an algorithm that will determine in a known and finite number of steps whether or not there is a solution and that yields a solution if there is one. However, problems of this type are NP-complete. In practical terms, this means that the implementation of these tests is considerably less straightforward than that for testing utility maximization (though not impossible, see [Cherchye et al. 2008](#)). As RP analysis is used to characterize data sets for different models, it is inevitable that some of these tests will face computational hurdles.

Contributions to the symposium There are six contributions to this symposium, which could be loosely divided into two groups.

(1): Three of the contributions to this symposium are directly related to Afriat's Theorem and two of them establish new connections between Afriat's Theorem and other fields of economics. Geanakoplos shows that there is a formal equivalence between the rationalizability problem and the problem of equilibrium existence for a particular zero-sum game; Afriat's Theorem can then be proved by appealing to von Neumann's equilibrium existence theorem for zero-sum games. Geanakoplos's approach to the problem also allows him to give a novel proof of another important and related RP result: The characterization, via the strong law of demand, of data sets that are rationalizable by quasilinear utility functions (For a direct proof of this result, see [Brown and Calsamiglia 2007](#)). Ekeland and Galichon's contribution to the symposium establishes a link between the rationalizability of consumer demand and the Pareto-efficient allocation problem known as the "housing problem" of [Shapley and Scarf \(1974\)](#). Just as departure from rationalizability in a consumer demand problem is characterized by cycles in the revealed preference relation, so departure from efficiency in an allocation problem is characterized by the existence of Pareto-improving trading cycles. Beyond establishing formal connections, these two papers are useful in allowing one to think

of the revealed preference problem in a different way and to import new tools for its investigation. In this connection, we should also mention the early paper of [Rochet \(1987\)](#), which shows that there is a connection between RP analysis and problems in mechanism design.

The third contribution to the symposium that is directly related to Afriat's Theorem is that of Forges and Iehl. In earlier work, [Forges and Minelli \(2009\)](#) have demonstrated that Afriat's Theorem does not in fact hinge on the linearity (in prices) of the budget sets. Even when budget sets are nonlinear, a suitably generalized version of GARP will guarantee that there is a continuous and monotone utility function that rationalizes the data (though the utility will typically not be concave). In their contribution, Forges and Iehl show that under an additional condition, which they call the "absence of contradictory statement," there exists a single utility function that *simultaneously* rationalizes almost all collections of budget sets with a given revealed preference pattern.

(2): Three contributions to this symposium also consider constrained choice problems but study issues that arise when the agent is not simply a utility maximizer. Building on the work of [John \(2001\)](#), Tvede and Keiding provide an RP analysis of a model of demand where the consumer has a complete but not necessarily transitive preference. In other words, we are in a situation where the consumer's preference relation has properties weaker than that considered by Afriat's Theorem. It is well known that when transitivity is dropped but completeness retained, the demand function will still obey the weak axiom of revealed preference though GARP may no longer hold; the more challenging RP issue, addressed by this paper, is in formulating the precise conditions that are sufficient for rationalizability within this more general class of preferences. Another contribution to this symposium, by Sandroni, Feddersen, and Cherepanov, gives an RP analysis of a model where the agent does not always choose according to his preference but could instead choose an "aspiration," provided it does not involve too great a loss in utility from his preferred choice. Models of this type are often used in modeling turnout at elections and public good contribution, but are sometimes criticized for being ad hoc; the RP analysis provides the model with a firmer foundation.

An issue closely related to the consistency of a data set with a particular model is the issue of *distinguishability*: Whether a data set can help the observer choose between two models or two related specifications of a general model. In certain situations, there may not, even in principle, be data sets that can distinguish between two models. An important question in the RP analysis of household models is whether it is possible to determine the public or private nature of a good within the household (without knowing the breakdown of consumption amongst household members), i.e., whether the good is divided up for personal consumption, whether it is used as a public good amongst household members, or whether there is an even more complicated arrangement. Cherchye, De Rock, and Platino's contribution to the symposium deals with this issue. They demonstrate that there are data sets consistent with one specification of a good's use within the household that is not consistent with another specification.

We are confident that this symposium will give the reader a good idea of some current trends in this vibrant research field, and we hope that it will stimulate new research on the topic.

References

- Afriat, S.: The construction of a utility function from expenditure data. *Int. Econ. Rev.* **8**, 67–77 (1967)
- Brown, D.J., Calsamiglia, C.: The nonparametric approach to applied welfare analysis. *Econ. Theory* **31**(1), 183–188 (2007)
- Brown, D., Matzkin, R.: Testable restrictions on the equilibrium manifold. *Econometrica* **64**(6), 1249–1262 (1996)
- Carvajal, A.: The testable implications of competitive equilibrium in economies with externalities. *Econ. Theory* **45**, 349–378 (2010)
- Carvajal, A., Deb, R., Fenske, J., Quah, J.K.-H.: Revealed preference tests of the cournot model. *Econometrica* (forthcoming) (2013)
- Cherchye, L., De Rock, B., Vermeulen, F.: The collective model of household consumption: a nonparametric characterization. *Econometrica* **75**, 553–574 (2007)
- Cherchye, L., De Rock, B., Sabbe, J., Vermeulen, F.: Nonparametric tests of collectively rational consumption behavior: an integer programming procedure. *J. Econom.* **147**, 258–265 (2008)
- Cherchye, L., De Rock, B., Platino, V.: Private versus public consumption within groups: testing the nature of goods from aggregate data. *Econ. Theory* (2013). doi:[10.1007/s00199-012-0729-8](https://doi.org/10.1007/s00199-012-0729-8)
- Cherepanov, V., Feddersen, T., Sandroni, A.: Revealed preferences and aspirations in warm glow theorem. *Econ. Theory*. (2013). doi:[10.1007/s00199-012-0733-z](https://doi.org/10.1007/s00199-012-0733-z)
- Chiappori, P.A.: Rational household labor supply. *Econometrica* **56**(1), 63–90 (1988)
- Ekeland, I., Galichon, A.: The housing problem and revealed preference theory: duality and an application. *Econ. Theory* (2013). doi:[10.1007/s00199-012-0719-x](https://doi.org/10.1007/s00199-012-0719-x)
- Forges, F., Iehle, V.: Essential data, budget sets and rationalization. *Econ. Theory* (2013). doi:[10.1007/s00199-012-0716-0](https://doi.org/10.1007/s00199-012-0716-0)
- Forges, F., Minelli, E.: Afriat's theorem for general budget sets. *J. Econ. Theory* **144**(1), 135–145 (2009)
- Fostel, A., Scarf, H., Todd, M.: Two new proofs of Afriat's theorem. *Econ. Theory* **24**(1), 211–219 (2004)
- Geanakoplos, J.: Afriat from MaxMin. *Econ. Theory* (2013). doi:[10.1007/s00199-013-0783-x](https://doi.org/10.1007/s00199-013-0783-x)
- Green, R.C., Srivastava, S.: Expected utility maximization and demand behavior. *J. Econ. Theory* **38**(2), 313–323 (1986)
- John, R.: The concave nontransitive consumer. *J. Glob. Optim.* **20**, 297–308 (2001)
- Keiding, H., Tvede, M.: Revealed smooth nontransitive preferences. *Econ. Theory* (2013). doi:[10.1007/s00199-013-0765-z](https://doi.org/10.1007/s00199-013-0765-z)
- Mas-Colell, A., Whinston, M.D., Green, J.R.: *Microeconomic Theory*. Oxford University Press, Oxford (1995)
- Quah, J.K.-H.: A Revealed Preference Test for Weakly Separable Preferences. Working Paper, Department of Economics, Oxford, No. 601 (2012)
- Richter, M.K.: Revealed preference theory. *Econometrica* **34**(3), 635–645 (1966)
- Rochet, J.C.: A necessary and sufficient condition for rationalizability in a quasi-linear context. *J. Math. Econ.* **16**(2), 191–200 (1987)
- Shapley, L., Scarf, H.: On cores and indivisibility. *J. Math. Econ.* **1**(1), 23–37 (1974)
- Varian, H.R.: The nonparametric approach to demand analysis. *Econometrica* **50**(4), 945–973 (1982)
- Varian, H.R.: Non-parametric tests of consumer behaviour. *Rev. Econ. Stud.* **50**(1), 99–110 (1983)
- Varian, H.: The nonparametric approach to production analysis. *Econometrica* **52**, 579–597 (1984)