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ORIGINAL ARTICLE

Learning curve constraint cell automaton model for the lean production of CFRP airframe components

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Abstract A new model is proposed for the lean production of carbon fiber reinforced plastic (CFRP) airframes. Our method improves the production rate by determining the ideal human-capital balance and inventory density on the factory line. The proposed model is derived as a two-step process: First, an analytical solution for the learning rate shift with a human-capital ratio is obtained by merging the Wright learning curve model into the Cobb-Douglas production function. Second, the solution is factored by an asymmetric simple exclusion process (ASEP) cell automaton model to assess whether the inventory density negates the theoretical learning effect. Recent moves toward lean production mean that aerospace CFRPs have a limited shelf life, minimizing buffer periods under metastable and stable production in the time discrete ASEP model. The shift from metastable to stable changes the production rate. Combined with the fact that ASEP is known to drastically reduce throughput if the production steps are not harmonized, the shipment probability p at each step becomes less than 1. Therefore, the human learning effect, which can alter the shipment rate, must be controlled so that $p = 1$ at each production step. This paper describes the analytical aspects of the apparent learning rate to determine adequate values for the human and capital resources, and thus harmonize the learning rates of the production steps. The analytical model shows that factory planning dominates the production rate of CFRP aerospace components. The model is applied to Boeing 787 production data, and it is found that a reduction in inventory density could improve the apparent delivery rate up to the maximum of the human potential.

Keywords CFRP · Airframe component · Cell automaton · Wolfram rule 184 · Asymmetric simple exclusion process (ASEP) · Lean production · Airframe production · Wright learning curve · Cobb-Douglas production function · Human fraction · Man-hour

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Nomenclature

ASEP:	Asymmetric Simple Exclusion Process
CA:	Cell Automaton
CFRP:	Carbon Fiber Reinforced Plastic
DOC:	Dream lifter Operations Center
LPC:	Lean Production Concept
TOC:	Theory of Constraints
p:	Shipment Probability
R:	Learning Rate
$P_i(1)$:	ith Production Factor in Operation
$P_i(0)$:	ith Production Factor in Idle
$TP_i(1)$:	Operational Time of ith Production Factor
$TP_i(0)$:	Idle Time of ith Production Factor
$B_i(1)$:	ith Buffer Factor in Operation
$B_i(0)$:	ith Buffer Factor in Idle
$TB_i(1)$:	Operational Time of ith Buffer Factor
$TB_i(0)$:	Idle Time of ith Buffer Factor
TR:	Throughput Ratio, defined as the throughput rate of the whole production chain divided by the rate of each production chain

ρ:	Work Density, defined as the input rate divided by the upper bound of throughput
$H(N)$:	Man-Hours of the N_{th} Product
C_1:	Man-Hour Reduction Rate
$\dot{N}$:	Production Rate at Nth Product
L:	Man-Hours of Human Activities
K:	Man-Hours of Automated Machines
C_L:	Man-Hour Reduction Rate in the Extreme Case where the Capital Fraction is 0
C_K:	Man-Hour Reduction Rate in the Extreme Case where the Capital Fraction is 1
C_2:	Adjustment Factor at a Fixed Time
C_3:	Partial Elasticity of Capital Input
C_4:	Partial Elasticity of Human Input

1 Introduction

Processes involving carbon fiber reinforced plastic (CFRP) components dominate the time path in the production of civil jet liners, because CFRPs are used in the main airframe structures such as wing boxes, fuselage barrels, and tail wings [1–5]. The critical chain of present airframe production is thus related to CFRPs from the cold storage of prepreg rolls through to the final setup for curing in autoclaves even though the low cost out-of-autoclave method has certain advantages over the autoclave process [6, 7]. Figure 1 depicts the schematic chain of CFRP production for wing panels.

Curing in an autoclave is a time-discrete process that has a constant throughput, with time-fixed factors such as the temperature gradients in heating and cooling, cure time, and the pressure gradients in vacuum degassing and high pressure compaction. Thus, the production chain is most effective when the autoclave is continually operating without any idling time.

From the basic concepts of the Theory of Constraints (TOC) [8–10], the constraint buffer for the inventories of CFRP green bodies is the key factor prior to the cure process in the autoclave (which is the bottleneck) for attaining maximum throughput in the whole production chain. However, the limited shelf life of recent CFRPs with toughened matrices leaves a minimal lead-time before the cure, meaning the whole of the production chain must adopt the lean production concept (LPC) [11].

LPC aims to shrink the timeline from cold storage to cure in autoclave by removing seven non-value-added types of waste:

1. Over-production of inventories that surpass the autoclave throughput.
2. Waiting time at the buffer before the cure.
3. Transportation between production steps, especially in the buffer zone.
4. Too much work in process before the buffer.
5. Unnecessary movement of workers from idling steps to normal operation.
6. Over-processing of inventories by idling workers, such as repeated checks.
7. Defective inventories, particularly that which has surpassed its shelf life.

An ideal no-waste LPC is attained when each process works with equal throughput toward the time-discrete autoclave, with a moving-line of controlled speed or takt time over the whole production line. The movement of inventories under an ideal LPC is mathematically identical to the metastable one-dimensional model of cellular automata (CA) and the Wolfram rule 184 one-dimensional CA array for the lower bound of the stable case [12–14]. Hence, an analytical approach is expected to provide a reasonable overview for the throughput of the production chain.

However, the learning effect does not always improve the throughput of each step when reducing the seven types of waste. Thus, we must control the learning effect to attain an equal improvement of throughputs throughout the process chain of the CFRP components. Wright was the first to report an exponential learning curve model, whereby the production per man-hour reduces by a constant 20 % (learning rate $R = 0.8$) with each doubling of accumulated airframe products [15]. Crawford unit factor theory is also expressed as an exponential-type learning curve, representing the cumulative man-hours of each airframe as the sum of the man-hours of each component [16]. These models have been extended for the case of experienced human resources, such as in the b-factor model [17, 18] and have been assessed using huge data sets for military aircraft from the Second World War and the Cold War [19, 20]. The relationship between the learning effect and cost reduction has been extensively discussed [21–23].

A certain uniformity of human activities is expected in the improvement curve. Various percentages have been reported, such as 80–85 % for the aerospace industry in general, with 70–80 % for civil engineering, 75–85 % for office work, and 78–80 % for the production of composite components [24–28].

This paper focuses on an analytical solution of the variable rate for the CFRP production process series. First, it is assumed that the reduction rate is a function of the partial production elasticity with respect to the labor input. Thus, variations in partial elasticity will affect the measured value, even though the human learning rate is uniform. Second, we adopt the Cobb-Douglas production function [29, 30] to merge the capital input with the Wright learning curve model. Third, the condition of the maximum production rate is applied to derive an analytical solution for the apparent

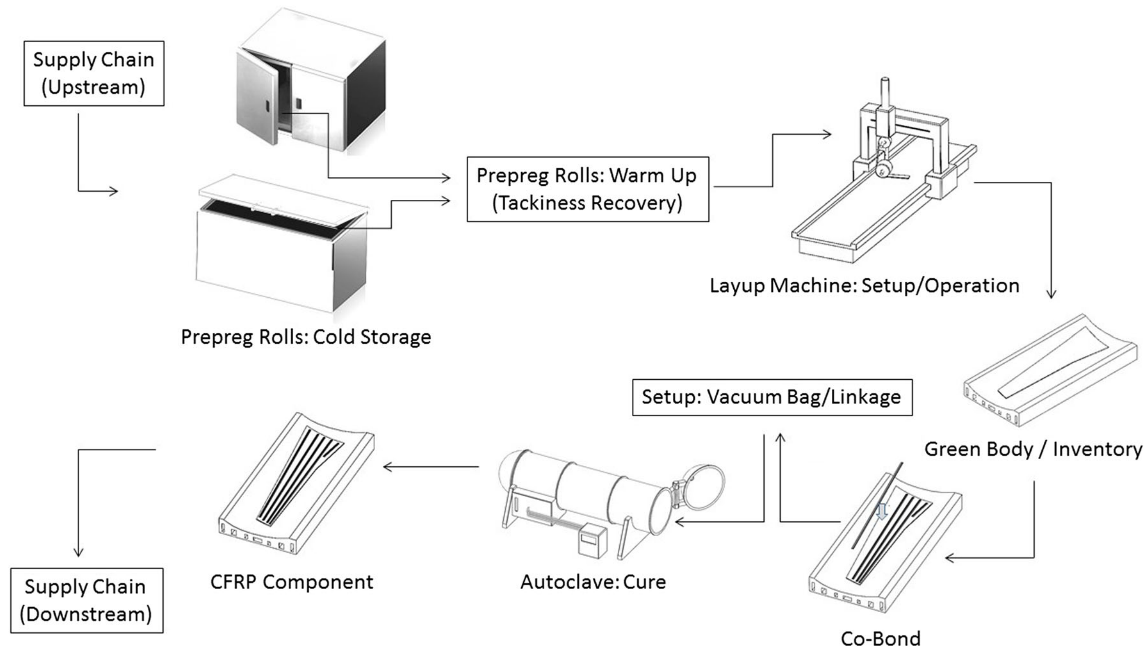


Fig. 1 CFRP production chain for a wing panel

learning rate in mass production. The solution is factored by a CA model of an extreme LPC case to assess an apparent learning effect of an ideal chain for CFRP components production.

2 Cell automaton model of CFRP lean production

Figure2 depicts the main chain of the production steps in Fig1. The ith Process/Buffer step in Fig.2 is represented in Fig.3, where $P_i(1)$ denotes an operational production factor, $P_i(0)$ is idle, $TP_i(1)$ denotes the operational time, and $TP_i(0)$ is the idle time. The buffer factor is represented as $B_i(1)$ operational and $B_i(0)$ idle, with corresponding times of $TB_i(1)$ and $TB_i(0)$.

The buffer may be depicted $B_i(n), n = 0, 1, 2,,$ as in Fig.4, for the case of numerous inventories.

A takt production line or a moving line of constant speed is represented as the array $(P_{i-1}, B_{i-1}), (P_i, B_i), (P_{i+1}, B_{i+1})$, as depicted in Fig.5.

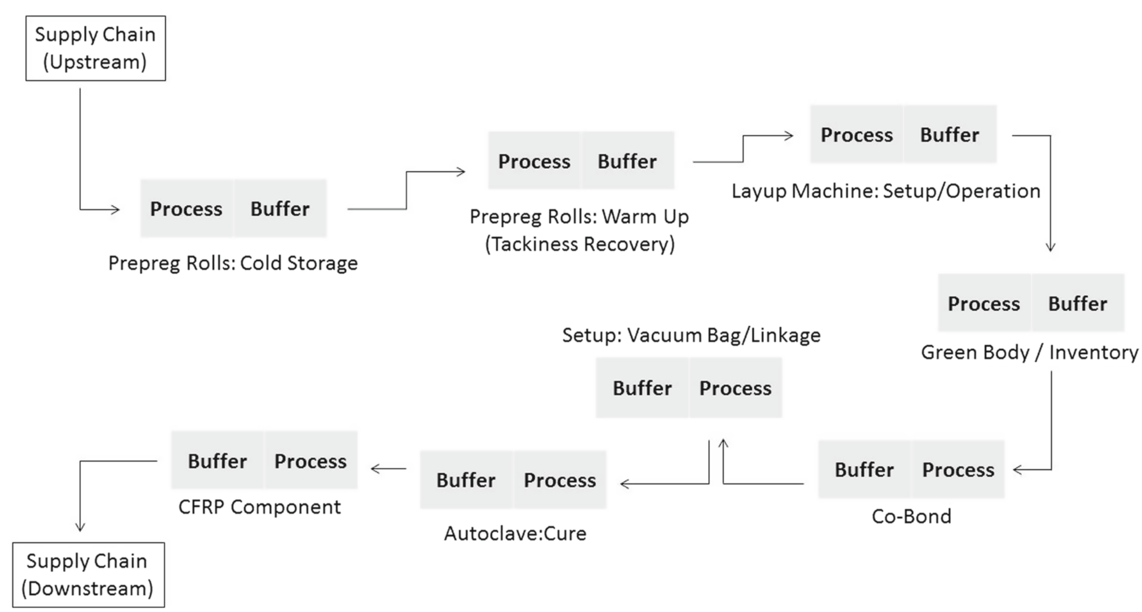
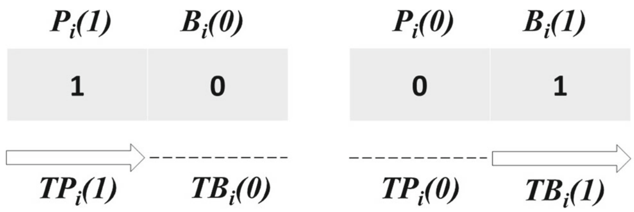
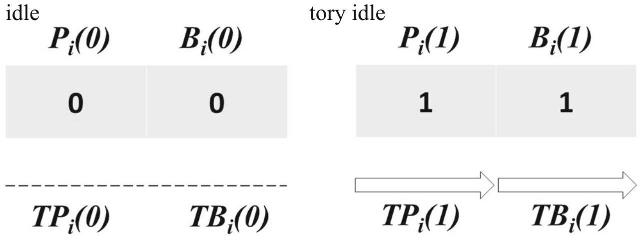


Fig. 2 CFRP production chain: schematic



(a) Process: operation, buffer: idle (b) Process: idle, buffer: inventory idle



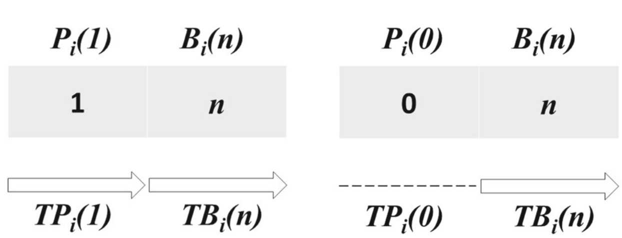
(c) Process: idle, buffer: idle (d) Process: operation, buffer: inventory idle

Fig. 3 ith process buffer step of CFRP production chain

$P_i(1)$ for the longest $TP_i(1)$ is the bottle neck, which constraints the throughput of the whole production chain under TOC. Thus, $B_{i-1}(n)$, $n \geq 1$, is the condition to avoid $P_i(0)$ and maximize the total throughput of the chain. The condition $\Sigma(TB_i(n)) \rightarrow min$, however, minimizes both the risk of surpassing the CFRP shelf life and the buffer cost, resulting in the LPC array $(P_{i-1}, B_{i-1}(0))$, $(P_i, B_i(0))$, $(P_{i+1}, B_{i+1}(0))$, as depicted in Fig.6.

Through chain elongation, the LPC in Fig. 6 approaches CA model of Wolfram rule 184, the one dimensional infinite array depicted in Fig. 7.

The one-dimensional CA of Wolfram rule 184 is expressed as an infinite array of cells that are limited to two possible states, 0 and 1. State 1 in a cell denotes that it moves to the right with a probability $p = 1$ at each time step when the right-hand cell is 0, whereas the movement is blocked if the right-hand cell has a value of 1. The case of $p < 1$ has



(a) Process: operation, buffer: nth inventory idle (b) Process: idle, buffer: nth inventory idle

Fig. 4 Large capacity buffer

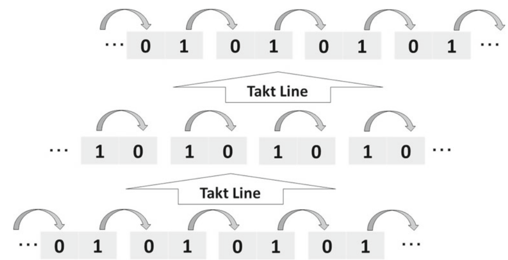


Fig. 5 Representation of CFRP production chain

been studied as an Asymmetric Simple Exclusion Process (ASEP), giving a solution for the throughput ratio

$$TR = \frac{1}{2} \cdot \left\{ 1 - \sqrt{1 - 4p\rho(1-\rho)} \right\} \tag{1}$$

where TR is the throughput of the whole production chain divided by the rate of each production chain for the periodical boundary condition depicted in Fig.8 [31–35]. The maximum throughput ratio $TR = 1/2$ is attained at a work density of $\rho = 1/2$ (work density is defined as the input rate divided by the upper bound of throughput) under the condition that probability $p = 1$ or a uniform $TP_i(1)$ is attained at any i, as depicted in Fig.9.

A metastable condition of $P_i(1)$ at any i, as depicted in Fig. 10, may theoretically duplicate the throughput, even though the slightest deviation of $TP_i(1)$ at any i negates the metastability and reverts to the stable ASEP model case of $TR \leq 1/2$. Thereby, the throughput of a finite array, which has a free end of $P(0)$, is distributed between the metastable upper bound and the lower bound of the ASEP infinite array model. To retain the metastable condition on the production line of a metal-based automobile, the Toyota Production System guarantees any worker at any step of the production chain the right to stop the whole production line to recover from any deviation. Thus, the apparent line operation is kept in this metastable condition. However, the

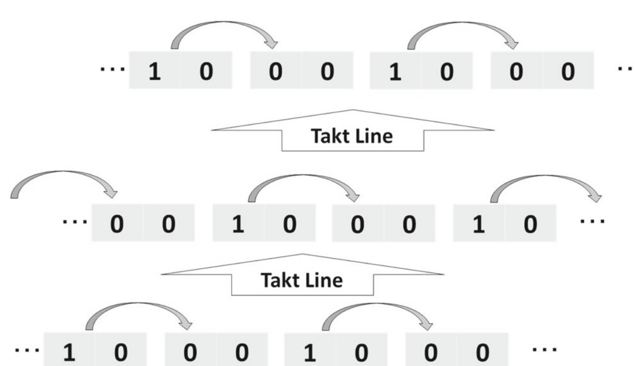


Fig. 6 Production chain of LPC

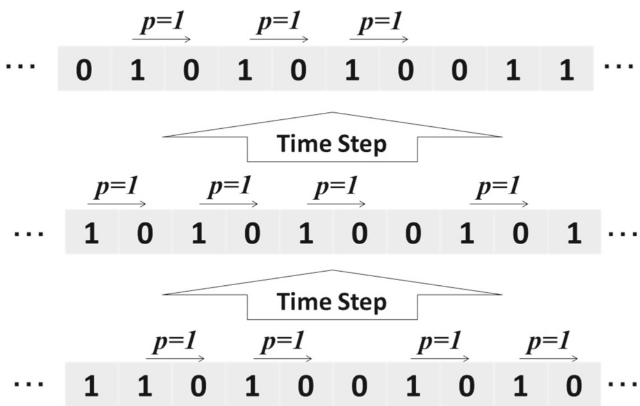


Fig. 7 Wolfram rule 184 CA model

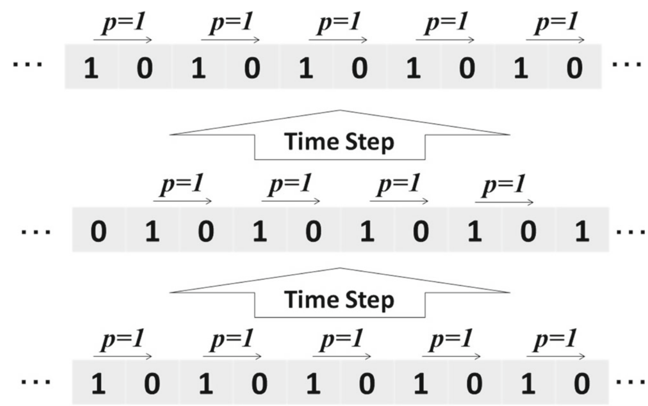


Fig. 9 Maximum throughput of ASEP model

limited shelf life of CFRPs means there is a risk of losing a huge inventory during this recovery period. In addition, airframes have over 10^6 components, considerably more than in automobiles (about 10^4). Thus, the airframe production line may stop 10^2 times more often than that of an automobile. Hence, the maximum throughput under the ASEP stable condition $TR = 1/2$ may be the ideal LPC target for CFRP airframe components.

The time trend of the metastable throughput in the ASEP model may exhibit a steep learning curve during the early production stage, before gradually flattening. The measured throughput may, however, be distributed between the upper bound of the learning curve in the metastable condition and the lower bound of the ASEP model, as depicted in Fig.11.

The lower bound of ASEP model attains the maximum throughput ratio $TR = 1/2$ when $\rho = 1/2$ and $p = 1$. However, an inadequate work density of $\rho \neq 1/2$ or troublesome chains of $p < 1$ lead to a reduced throughput ratio of $TR < 1/2$, as depicted in Fig. 12.

An excess input of inventory may lead to $\rho > 1$, causing the throughput ratio to tend to $TR = 0$. Thereby, the lower bound curve shifts rightward until the learning effect of *upper bound throughput = input rate*; thus, $\rho = 1$ as depicted in Fig. 13. Above all, the throughput becomes distributed in such a way that *metastable upper bound* $\geq$ *measured throughput* $\geq$ *stable lower bound of TR* $\times$ *metastable upper bound*. Thus, a uniform $TP_i(1)$ at any i is the key factor in maximizing the throughput as $TR \to 1$. However, the learning effect that is apparent in airframe production does not always improve TR equally throughout the production chain. Thus, an analytical approach for the learning effect is needed, and so we seek the parameters that will produce an equal improvement throughout the production chain.

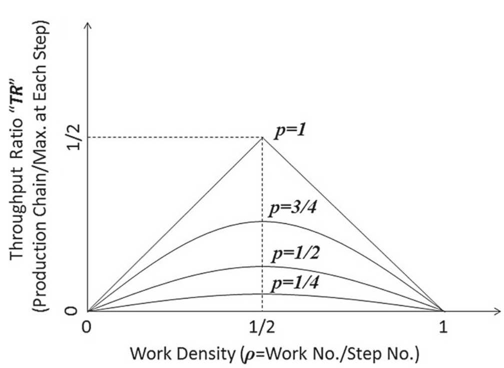
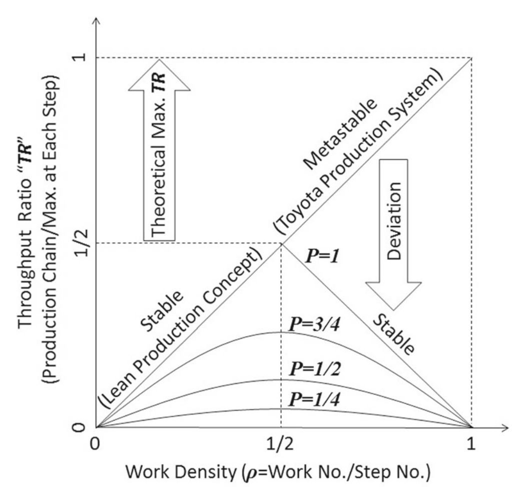
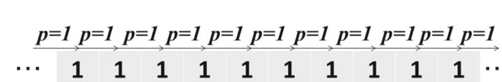


Fig. 8 Throughputs of ASEP



(a) Maximum throughput at metastable condition



(b) CA model of metastable condition at $\rho = 1$

Fig. 10 Toyota production system

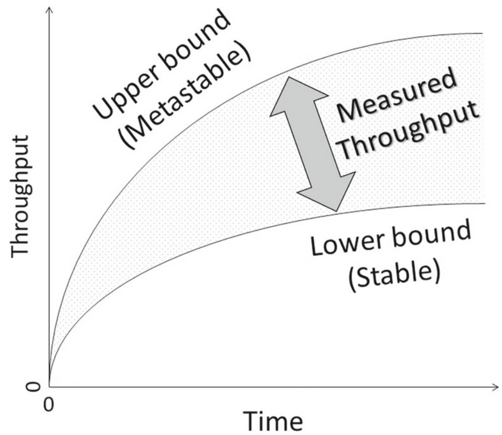


Fig. 11 Time trend of throughput

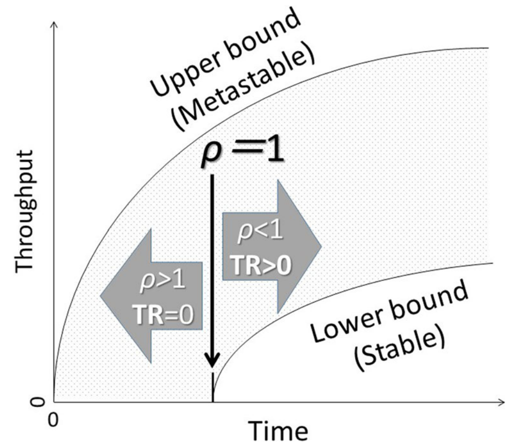


Fig. 13 Lower bound shift due to excess work input

3 An analytical solution for the learning rate

3.1 Reported learning rates

Wright reported that the reduction in cumulative man-hours could be expressed for each airframe on a fixed production line as follows:

$$H(N) = H(1) \cdot N^{-C_1} \tag{2}$$

where $H(N)$ denotes the man-hours of the N_{th} product and C_1 is the man-hour reduction rate ($0 \leq C_1 \leq 1$). Figure 14 illustrates the trend in man-hour reduction, or learning-curve, of Eq. 2.

Writing the learning curve rate R as in Eq. 3

$$R \equiv \frac{H(N)}{H(2N)}, \tag{3}$$

we can express C_1 as:

$$C_1 \equiv -\frac{\ln(R)}{\ln 2}. \tag{4}$$

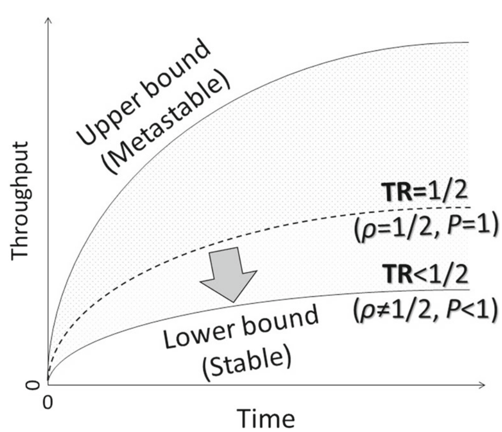


Fig. 12 Lower bound of reduced throughput ratio

The production rate $\dot{N}$ may be inversely proportional to the man-hours. Thus, the productivity may be expressed with the Wright learning curve as follows:

$$\dot{N} = \dot{N}_{N=1} \cdot N^{C_1} \tag{5}$$

Figure 15 illustrates the trend of improved productivity.

3.2 Assumptions for the learning rate analysis

Following the work of Wright, various values of R have been reported: $R = 0.7 - 0.8$ in civil engineering, $R = 0.75 - 0.85$ for office work, and $R = 0.78 - 0.80$ in the production of composite components. These values may imply that:

1. the simple repetition of 100 % human actions leads to $R = 0.7$, and thus $C_1 = 0.515$,
2. $R = 1.0$, and thus $C_1 = 0$ for unmanned, 100 % automated production, and
3. the human fraction is given by some parameter of Rs.

Thus, the following assumptions are made:

1. $R = 1.0$ in the extreme case when the human fraction of man-hours $L/(L + K)$ is 0 or $K/(L + K) = 1.0$,

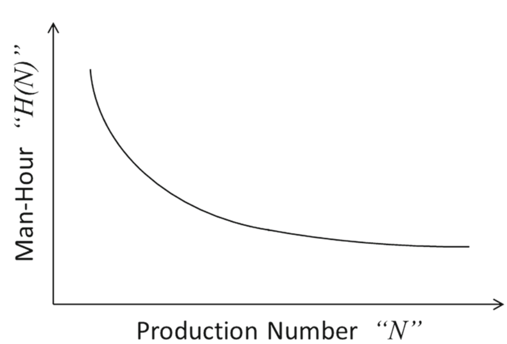


Fig. 14 Learning curve

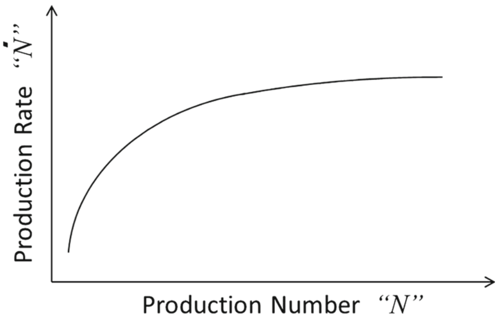


Fig. 15 Improvement in productivity

where L denotes the man-hours of human activities and K denotes those of automated machines or capital.

2. The total man-hours capacity of the airframe line $L+K$ is constant for the first airframe $H(1)$, i.e.,

$$H(1) = L + K \tag{6}$$

3. An effective C_1 is set as the sum of the fractions given by K and L as follows:

$$C_1 \equiv \frac{C_L \cdot L + C_K \cdot K}{L+K} = C_L \cdot \frac{L}{H(1)} \tag{7}$$

where C_L is the man-hour reduction rate in the extreme case where the capital fraction is 0 or $H(1) = L$. In this scenario, $C_L = 0.515$ by assumption (1) and C_K, which is the man-hour reduction rate for $H(1) = K$, is 0 by assumption (2).

3.3 A Known production function for variable human-capital fractions

The Cobb-Douglas production function, which is widely used in economics, relates the throughput rate to the inputs as:

$$\dot{N} = C_2 \cdot K^{C_3} \cdot L^{C_4} \tag{8}$$

where $\dot{N}$ is the production rate, C_2 is an adjustment factor at a fixed time or N, and C_3, C_4 are the partial elasticity of capital and human inputs, respectively. The case of constant returns to scale is assumed in the following analysis as the sum of the partial elasticities is equal to 1.

$$C_3 + C_4 = 1 \tag{9}$$

This assumption may be rational for aircraft production, as λ times $K + L$ or λ times the number of production lines leads to an increase in $\dot{N}$ by a factor of λ. Cobb and Douglas reported that $C_3 = 1/4$ and $C_4 = 3/4$. Figure 16 shows the schematic Cobb-Douglas-type productivity curve for the case where $K + L = Const$.

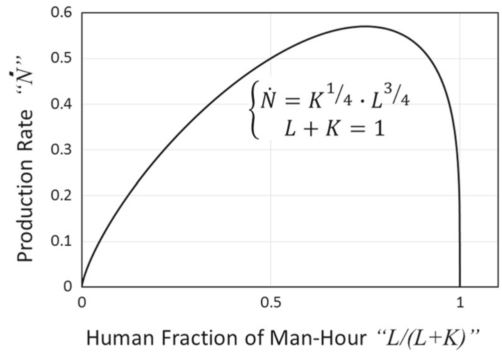


Fig. 16 Cobb-Douglas-type productivity curve

3.4 A new production function for variable learning rates

The production functions in Eq. 5 and Eq. 8 are known to provide a good fit with production data for various parameters. Thus, the independent 2D schematics in Figs. 15 and 16 imply a "3D productivity surface,"as shown in Fig. 17.

Thus, Eqs. 5 to 9 can be written in functional form as:

$$\dot{N} = C_5 \cdot \left\{1 - \frac{L}{H(1)}\right\}^{C_3} \cdot \left\{\frac{L}{H(1)}\right\}^{(1-C_3)} \cdot N^{C_L \cdot \frac{L}{H(1)}}, \tag{10}$$

where C_5 is an adjustment factor. The maximum production rate for the first airframe, where $N = 1$, is given under the condition $(\partial \dot{N}/\partial N)_{N\to 1} = 0$. Thus,

$$L = (1 - C_3) \cdot H(1). \tag{11}$$

Thereby, Eq. 2 and Eq. 11 provide the reduction rate C_1 and the production rate $\dot{N}$ as follows:

$$\begin{cases} C_1 = (1 - C_3) \cdot C_L \\ \dot{N} = \dot{N}_{N=1} \cdot N^{(1-C_3)\cdot C_L} \end{cases}. \tag{12}$$

Equations 11 and 12 are important to demonstrate that the ideal human-capital balance is equal to the partial elasticity ratio of the capital and human inputs when planning an isolated new production line. For example, when $C_L = 0.515$

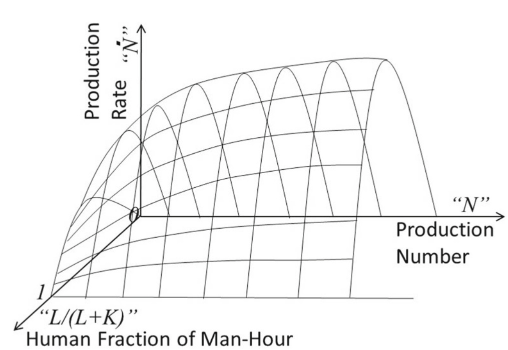


Fig. 17 Expected productivity surface

and $C_3 = 1/4$, the ideal reduction rate for a new production line is expected to have:

$$\begin{cases} C_1 \approxeq 0.386 \\ R \approxeq 0.765 \end{cases} . \tag{13}$$

When this new line is not isolated, and workers are able to move between lines, modulating the human fraction $L/(L+K)$ may maximize the production rate of the new line for the case $N > 1$, giving $\partial \dot{N}/\partial L = 0$ as follows:

$$\left\{\frac{L}{H(1)}\right\}^2 + \left\{\frac{1}{C_L \cdot \ln(N)} - 1\right\} \cdot \frac{L}{H(1)} - \frac{1-C_3}{C_L \cdot \ln(N)} = 0 \tag{14}$$

$$\therefore \frac{L}{H(1)} = \frac{1}{2} \cdot \left[-\left\{\frac{1}{C_L \cdot \ln(N)} - 1\right\} + \sqrt{\left\{\frac{1}{C_L \cdot \ln(N)} - 1\right\}^2 + \frac{4 \cdot (1-C_3)}{C_L \cdot \ln(N)}} \right] \tag{15}$$

as $L \geq 0$.

$$\therefore C_1 = \frac{C_L}{2} \cdot \left[-\left\{\frac{1}{C_L \cdot \ln(N)} - 1\right\} + \sqrt{\left\{\frac{1}{C_L \cdot \ln(N)} - 1\right\}^2 + \frac{4 \cdot (1-C_3)}{C_L \cdot \ln(N)}} \right] \tag{16}$$

4 Discussions

4.1 Human fraction and productivity improvement

Wright assigned from the production data of a twin-sheet single engine airplane, giving $C_1 \approxeq 0.322$. This parameter implies that a 50 % reduction in man-hours can be attained for every factor of 10 increase in production, and a 90 % cut is attainable through a 1000 times production increase. However, Eq. 13 implies that an improved production rate can be achieved by adding more human resources so that $R = 0.765$ or $C_1 \approxeq 0.386$ when $C_L = 0.515$. Furthermore, increasing skilled human resources according to Eq. 15 can drastically improve productivity. Figure 18 shows the Wright production curve with $R = 0.8$ or $C_1 \approxeq 0.322$, the fixed human fraction case with $R = 0.765$ or $C_1 \approxeq 0.386$ (approximately 40 % improvement over the Wright production curve), and the variable human fraction case of Eq. 15.

This analysis has revealed that practical inputs of well-trained human resources result in drastically improved productivity. Figure 19 shows the human fraction of Fig.18. This implies that (1)Wright could attain drastically improved productivity if 60 % more man-hours could be assigned to skilled workers; (2)long-term contracts may be more reasonable, both for airframe companies and workers, as fixing $L = 0.75$ provides the optimal productivity in the Wright type learning curve model; (3)the maximum productivity occurs when new, skilled workers continuously join the production line.

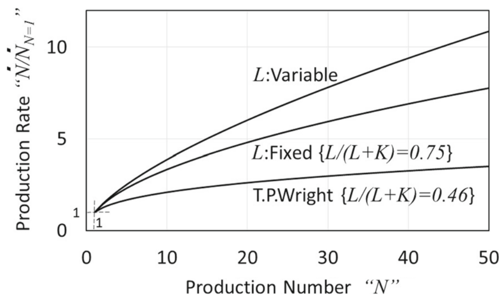


Fig. 18 Productivity improvement for airframes

4.2 Potential production rate of CFRP airframes: the case of Boeing 787

The Boeing 787 was the first commercial jetliner to adopt both the large-scale application of CFRP for the primary structure (up to 50 % wt.) and LPC across its worldwide supplier network [36–38]. The CFRP wing box, one of the primary structures, is produced in Nagoya, Japan. The first autoclave for the cure of CFRP wing panels was completed in 2006, and the first product was shipped in May 2007; a second autoclave was added in 2011, and the 100th product shipped in December of 2012. The shipment buffer Dream lifter Operations Center (DOC) has been in operation since March, 2014 [39–45].The stringer production sub-chain in Yamaguchi, Japan, was reinforced in 2014 to increase wing box shipments from the current 10 units per month to 14 by 2016 [46]. The final assembly sites in Everett and North Charleston, USA, made their first commercial deliveries to All Nippon Airlines (ANA) in September, 2011 [47].

The delivery trends, except for early-stage prototypes and test models, are depicted in Fig. 20 (Retrieved form the Boeing Company Website [48]). Monthly delivery numbers vary from 0 in the early production stages to 15 by mid-2014.

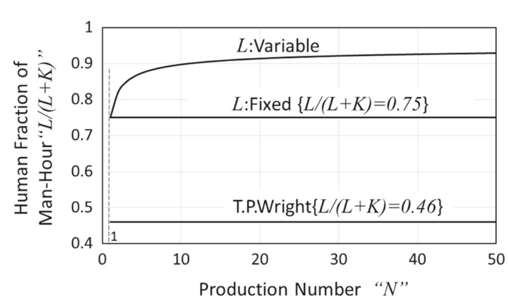
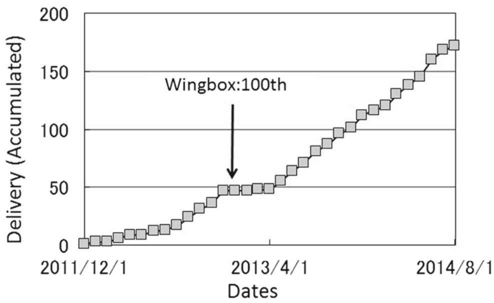
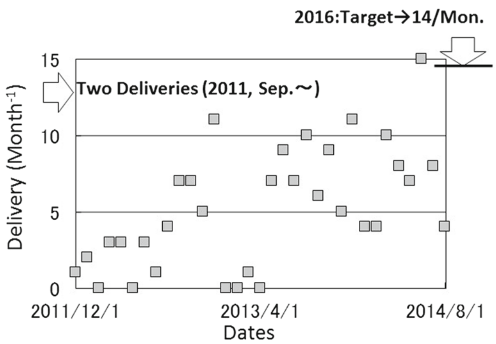


Fig. 19 Human fraction of airframe production



(a) Accumulated delivery



(b) Monthly delivery

Fig. 20 Delivery trends for the Boeing 787

Deviations in the production rate during the early production stages steadily converged from 2011 to 2012. However, a marked increment during a production campaign to reach 10 deliveries per month (and which rewarded engineers with an 8 % bonus) [49, 50] was followed by stagnation to 0 deliveries per month from late 2012 into 2013.

The metastable curve given by Eq. 2 suggests a learning curve rate $R = 0.7$, as depicted in Fig. 21.

Thus,

$$\dot{N} = \dot{N}_{N=1} \cdot N^{0.515}, \tag{17}$$

where N is the real cumulative production number. A value of $R = 0.7$ (or $C_1 = 0.515$) implies, from Eq. 12, that the human fraction contributing to the production is almost

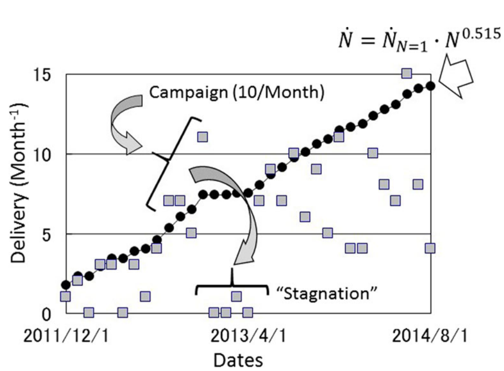
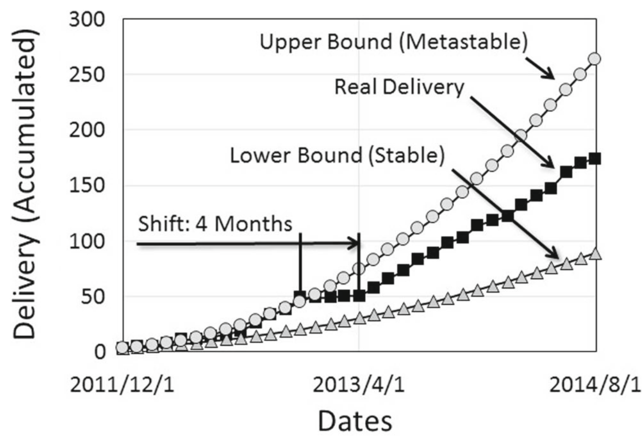
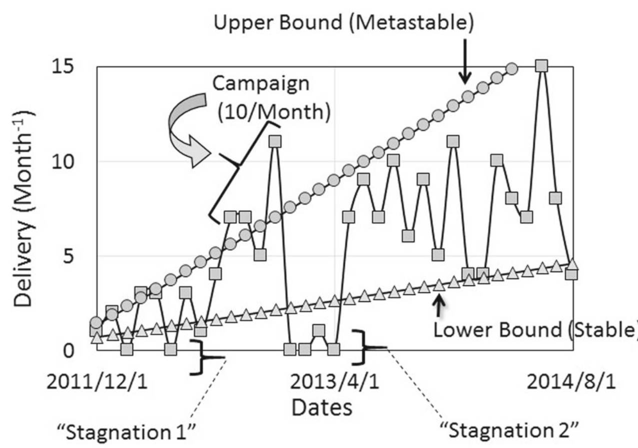


Fig. 21 Expected metastable delivery rate

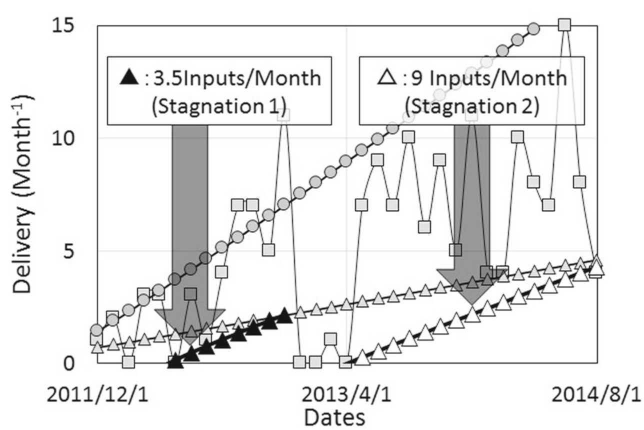
100 %. This is possibly due to the human operations at each step for quality inspections and recovering from errors in automated processes. Thereby, we can deduce that the lean production of Boeing 787s is highly complex, and thus the inclusion of skilled engineers is the key factor for steady throughput.



(a) Accumulated deliveries



(b) Monthly delivery



(c) Lower bound shift due to excess input

Fig. 22 Expected deliveries at metastable and stable production rates

In addition, artificial changes to the learning trend may even negate the throughput improvement, as evidenced during the campaign periods. This can be explained by overtime resulting in excessive inventory. The maximum production case is given by Eq. 17, as illustrated in Fig. 22a. This reveals that the real production trend was close to the maximum before the 10-per-month production campaign, whereas afterward the trend had simply shifted right by 4 months to provide a new trend between the stable lower bound and the metastable upper bound.

The monthly production rates in Fig. 22b show the two stagnation periods: (1) during the early production stage and 2) just after the production campaign. The two stagnations may be explained by the case shown in Fig. 13, where the inventory inputs have exceeded the work density capacity, i.e., $\rho \geq 1$. Figure 22c depicts the fixed-input case of 3.5 inventories per month during the early production stage and 9 following the campaign. This can explain the stagnations and shifts in the lower bounds and implies that two inputs would have been ideal in the early stage to keep $\rho = 1/2$. A steady increment from five inputs to 10 during the periods from 2013/4/1 to 2014/8/1 could have improved the delivery rate by avoiding the second stagnation.

In addition, Fig. 22 implies that the workers had the potential to have achieved 14 deliveries per month in 2014 (the target for 2016) under ideal management in the metastable condition. Furthermore, the real delivery trend shows that moderate management should enable the 14 deliveries per month target to be reached in 2016, while the worst case of the lower bound implies this could take until late 2020. Thus, we believe effective line management will maximize the learning effect and minimize stagnation. This is the key to business success with CFRP aircraft: tuning the line parameters of the human-capital balance and the inventory density.

5 Concluding remarks

An analytical model has been derived for the learning rate in airframe production by extending the Wright learning curve model of reported learning rate and the Cobb-Douglas production function for variable human-capital fraction. In addition, deviations in the real delivery rate from the ideal case have been explained using an ASEP CA model as stagnation resulting from excessive work. Important findings stress the need to identify: (1) adequate human resource inputs, which maximize the learning rate, both for planning an isolated production line and lines that workers are able to move between, (2) optimum work density for maximizing the throughput, and (3) a stagnation trigger density that negates the learning effect.

The model has been applied to Boeing 787 production data from 2011/12/1 to 2014/8/1to have implied that (1) the human fraction for the contribution to the production rate improvement is almost 100 %, (2)the stagnation to 0 deliveries in the early production stage had been explained by the excessive input of 3.5 inventories per month while two would have been ideal to improve the delivery late, and the 4 months stagnation around 2013/4/1 is explained by the excessive input of nine inventories per month while a steady increment from 5 to 10 during the periods from 2013/4/1 to 2014/8/1 would have been ideal, (3)the workers had the potential in the metastable production condition to have achieved 14 deliveries per month in 2014. Thereby, the model can help the planning for the maximum productivity of airframe manufacture throughout the commercial life of a plant.

An idealized full production has been assumed in the analysis, while repressed delivery rate and sudden cancelation are usual, especially for military aircraft. Thus, economic crises and unstable defense policies may cause the delivery rate to deviate from the estimation. In addition, workers of hypothetical learning effect have been assumed, while appropriate education and training programs can drastically improve the apparent learning effect. Furthermore, reliable verification data have been found only for Boeing 787. Therefore, we hope to obtain production data for other CFRP aircrafts, which have been in full production following the Boeing 787, such as the Boeing 777-X, Airbus A350, Bombardier C Series, Mitsubishi MRJ, Lockheed-Martin F35, and the US Air Force T-X.

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