Dynamical changes during composite milling: recurrence and multiscale entropy analysis

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Abstract Fluctuations describing chatter in the milling process of a composite material were analyzed using statistical, recurrence, and multiscaled entropy analyses. Through changing the rotational speed, we observed the appearance of chatter vibrations and signatures of intermittency. The corresponding characteristic change of recurrences could be used to invent a new efficient control procedure of milling. The workpiece was prepared from nonuniform material based on epoxy-polymer matrix composite reinforced by carbon fibers.

Keywords Nonlinear oscillations · Noise · Milling process · Composite material · Recurrence plot · Multiscaled entropy

1 Introduction and experimental data

The machining technology is the most important component in the modern massive production. Over the past years, its fast development gave way to a reliable high-speed cutting procedure. Consequently, elimina-

tion and stabilization of the associated chatter oscillations have become a high interest in science and technology [1–7]. The plausible adaptive control concept, based on relatively short time series, is studied to gain deeper understanding.

On the other hand, fiber reinforced composites are widely used in various applications due to their high specific strength and stiffness. In fact, many products are made in almost expected shapes. Thus, some machining has to be performed to meet more detailed requirements and assembly needs. Material discontinuity, nonhomogeneity and anisotropy in various products make machining of composites more challenging than machining simple metals and their alloys.

Due to various possible damage mechanisms (such as fiber pullout, fiber fragmentation and delamination, matrix burning, and/or cracking), poor cut surface quality can occur [8, 9].

The present measurements were conducted using the experimental set-up, shown in Fig. 1. Figure 1 illustrates the schematic diagram together with the workpiece surface image and a photo of the face cutter. The set-up was composed of a CNC milling machine, a piezoelectric dynamometer for cutting forces measurement, a charge amplifier, a module for simultaneous sampling, and a typical analog–digital converter. The cutting force signals were transmitted from the dynamometer to the analog–digital converter and finally to the computer system. Machining of epoxy-polymer matrix composite reinforced by carbon fibers was performed for various rotational speeds ranging from 2,000 to 8,000 rpm and the fixed feed 520 mm/min. Whereas, the depth of cut was equal to 0.5 mm, the mill itself was made of a diamond-coated cutting steel with a diameter of 12 mm.

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Fig. 1 Scheme of the experimental set-up and a view of the tool and the workpiece

2 Embedding and recurrence plots analysis

To perform recurrence plot analysis, we had to reconstruct a phase space from single observation ($x = F_x$). Following Takens theorem [10], the state of the system can be represented by the discrete time delay vector

$$\mathbf{x}_i = [x_i, x_{i-\Delta i}, x_{i-2\Delta i}, \dots, x_{i-(m-1)\Delta i}], \quad (1)$$

where m denotes embedding dimension and Δi is the corresponding time delay in sampling interval units (the sampling frequency was 4,000 Hz). Looking for minimum in the average mutual information to determine the Δi we obtained 7, 8, 6, 4, and 7, respectively, for increasing spindle speed (2,000, 3,500, 5,000, 6,500 to 8,000 rpm) [11]. In further analysis, we estimated the embedding delay for the system to $\Delta i = 7$. The embedding dimension was fixed to 6 with the help of minimizing the fraction of false neighbors [11–13].

After reconstruction of the phase space, we calculated the Euclidean distance matrix $\mathbf{R}$ using the independent time-delayed coordinates:

$$\mathbf{R}_{ij} = \Theta(\epsilon - \|\mathbf{x}_i - \mathbf{x}_j\|), \quad (2)$$

where ϵ was a threshold value which has a meaning of the tolerance of recurrence and $\Theta(\cdot)$ was the Heaviside step function. The distance matrix consisted of zeros and ones and corresponded to the state of the system (1—recurrence and 0—no recurrence). Below, one can see the graphical representation of the distance matrix called recurrence plot (RP) [14].

The recurrence plot method is extended into quantities based on points and lines in RP. One of the quantities, the recurrence rate (RR), measures the ratio between recurrence points and all possible points on the graph [15–17]:

$$RR = \frac{1}{n^2} \sum_{i,j=1}^n R_{ij}. \quad (3)$$

Another quantity, determinism (DET), is based on lines parallel to the main diagonal [15–17]:

$$DET = \frac{\sum_{l=l_{\min}}^n lP(l)}{\sum_{l=1}^n lP(l)}, \quad (4)$$

where $P(l)$ is a histogram of diagonal lines of the length l and the minimum length of lines is defined by $l_{\min} = 2$. One can use determinism to establish predictability of the dynamic of the system. Thus, the larger value of DET the more predictable system with diagonal lines in RP. Each line parallel to the main diagonal can be interpreted as a recurrence of two different states of

Fig. 2 The cutting force component F_x for different values of excitation speed: $\omega = 2,000, 3,500, 5,000, 6,500, 8,000$ rpm (a–e)

the system for the time equal to the length of the line. In analogy to determinism, the laminarity (LAM) is defined for vertical line patterns:

$$\text{LAM} = \frac{\sum_{v=v_{\min}}^n v P(v)}{\sum_{v=1}^n v P(v)}, \quad (5)$$

where $P(v)$ denotes a histogram of vertical lines of the length v with the minimum line length $v_{\min} = 2$. The larger the laminarity parameter, the more stable behavior of the system. In a limit of the laminar motion, the dynamical attractor of the examined system is unchanged topologically.

Fig. 3 Recurrence plots for $\epsilon = 0.4$ and different speed values ($\omega = 2,000, 3,500, 5,000, 6,500, 8,000$ rpm in (a–e), respectively)

The recurrence quantification analysis (RQA) [15–17] contains some more indicators based on diagonal and vertical lines. It provides us with the probability $p(l)$ or $p(v)$ of line distribution according to their lengths l or v (for diagonal and /or vertical lines). They are calculated

$$p(x) = \frac{P^\epsilon(x)}{\sum_{x=x_{\min}}^N P^\epsilon(x)},$$

$$(6) \quad L_{\text{ENT}} = - \sum_{l=l_{\min}}^N p(l) \ln p(l). \quad (7)$$

where $x = l$ for vertical structures in the specific recurrence diagram. $P^\epsilon(x)$ denotes the histogram of x and a fixed value of ϵ . Basing on distributions $p(x)$ the Shannon information entropy (L_{ENT}) can be defined for diagonal line collections

Fig. 4 Recurrence plots for $\epsilon = 0.6$ and different speed values ($\omega = 2,000, 3,500, 5,000, 6,500, 8,000$ rpm in (a–e), respectively)

Table 1 Summary of statistical analysis and recurrence quantification analysis (RQA) ($m = 6$, $\Delta i = 7$, and $w = 1$)

Frequency of spindle (rpm)	Average $\bar{F}_x(N)$	SD $\sigma_{F_x}(N)$	Kurtosis K	Recurrence rate ($\epsilon = 0.4, 0.6$)	Determinism ($\epsilon = 0.4, 0.6$)	Laminarity ($\epsilon = 0.4, 0.6$)	entropy L_{ENT} ($\epsilon = 0.4, 0.6$)	L_{mean} ($\epsilon = 0.4, 0.6$)
2,000	6.49	22.67	2.753	0.00006, 0.00058	0.033, 0.059	0.0184, 0.0798	0.950, 0.461,	3.200, 2.205
3,500	11.56	24.88	3.074	0.00012, 0.00075	0.265, 0.251	0.0021, 0.0447	0.901, 1.414	2.465, 3.300
5,000	12.05	21.99	2.929	0.00007, 0.00059	0.031, 0.133	0.0018, 0.0123	0.901, 0.725	2.125, 2.322
6,500	7.70	29.80	3.239	0.00006, 0.00045	0.119, 0.198	0.0000, 0.0014	0.377, 1.088	2.333, 2.697
8,000	9.54	20.92	2.541	0.00125, 0.00249	0.742, 0.720	0.0000, 0.0077	1.883, 1.826	7.683, 10.126

The rise in L_{ENT} signals the increase of complexity of considered time series. Finally, the average length of the diagonal lines L_{mean} :

$$L_{mean} = \sum_{l=l_{min}}^N l p(l) \quad (8)$$

is a parameter indicating the system stability (Fig. 2).

The results of RP calculations are shown in Figs. 3 and 4 for $\epsilon = 0.4$ and 0.6 , respectively. Note, we use the limit of small ϵ to apply the same embedding in exploring the topological patterns of different cases (Table 1). To compare the results of different spindle speeds, in further calculations, we use the normalized time series by subtracting their averages $\bar{F}_x$ and dividing by the corresponding values of square deviation σ_{F_x} (Table 1). In both figures one can see similar tendencies. The higher the spindle speed, the more regular features in RP. The above indicates that the points are located on specific lines. In Figs. 3e and 4e, more of the empty off-diagonal regions are observed for the highest speed (8,000 rpm) which can be interpreted as the loss of stationarity or the switch in the presence of additional modulation. Note, both interpretations converge for short time series pieces. For better clarity, we have estimated the RQA parameters: RR, DET, LAM, L_{ENT} , and L_{mean} . They are presented in Table 1, together with the other statistical parameters as the average value of force F_x , its square deviation σ_{F_x} and kurtosis K . Note, the average F_x is growing with the increasing spindle

speed $\omega = 2,000$ – $5,000$ rpm and after sudden decrease in 6,500 rpm, it becomes again larger for 8,000 rpm. The ω dependence of amplitude measured by the standard deviation is different reaching two maxima for 3,500 and 6,500 rpm. This could mean that there is some region of instability or chatter-related resonance at the speeds $\omega = 3,500$ and 6,500 rpm.

As far as the kurtosis is concerned, in most cases, we observe its variation with the increasing spindle speed (Table 1). Thus F_x distributions change from more uniform ($K < 3$) in Fig. 5a, c, and e to more peaked ($K > 3$) in Fig. 5b, d. Note, the larger kurtosis can indicate the occurrence of bifurcation or intermittency [18, 19]. The possibility of a bifurcation is signaled by the distribution of the F_x force (Fig. 5) manifesting the second peak for the fairly low rotational frequency.

In terms of RP parameters (for $\epsilon = 0.4$ and 0.6 in Table 1) we also observe the considerable increases of RR and DET for $\omega = 3,500$. The case $\omega = 8,000$ rpm seems to be important. However, this effect could be related to the same choice of the embedding. The embedding which we use is more relevant for lower speeds as the sampling frequency is the same for all cases. For the case of $\omega = 3,500$ rpm, we see the interesting local increases for L_{ENT} and L_{mean} for $\epsilon = 0.6$. These are difficult to interpret because entropy is usually related to the increase of complexity (wider distribution diagonal line lengths) whereas the increase of L_{mean} could be related to the reduction of exponential instability. The inverse of L_{mean} (or L_{max} for longer time series) used to

Fig. 5 The measured F_x distribution of the examined cases (Fig. 2). The arrows show the creation of the second peak present for the low rotational frequency

be discussed as the parameter scaling the Lyapunov exponent [20]. Thus, the larger entropy and smaller Lyapunov divergence indicate the opposing effects. These simple contradictions can be eliminated only for the bifurcation point. Looking more carefully into Fig. 4b one can clearly identify the characteristic dark squares discussed earlier as the evidence of intermittency [21–23].

Additionally, RR and DET were plotted versus the threshold value ϵ in Fig. 6a and b. The DET results indicate strongly nonmonotonic behavior. This effect, as a moderate size peak, is appearing in the case of ω of 3,500 rpm and develops fully in the case of 8,000 rpm.

The above results are leading us to the conclusion that the milling process can be identified by the recurrence plots approach. Previously, a similar approach was used successfully to distinguish blunt and sharp knives in the rock cutting by ripping head power [22] as well as to identify the cracks in vibrating plates and rotors [24, 25]. The RQA approach was also successfully

used to detect transient behavior in the milling process [26].

3 Multiscaled entropy analysis

The milling process as many other physical systems evolves on multiple temporal and spatial scales and is governed by complex dynamics. In recent years there has been a great deal of interest in quantifying the complexity of these systems. However, no clear and unambiguous definition of complexity has been established in the literature [27]. Going beyond the intuitive “structural richness” several notions of entropy have been introduced to describe complexity. Costa et al. [28] points out, that the traditional entropy measures quantify the regularity (predictability) of the underlying time series on a single scale, and there is no direct correspondence between regularity and complexity.

For complex systems with multiple temporal or spatial scales, a definition of complexity should include the described below multiscale features. Recently, Costa et al. [29] introduced the concept of multiscale entropy (MSE) to describe the complexity of such multiscale systems. They successfully used this concept to describe the nature of complexity in physiological time series such as those associated with cardiac dynamics [30] and gait mechanics [29]. In this section we perform a multiscale entropy analysis as a measure of complexity in cycle-to-cycle variations of milling force component F_x . MSE is based on a coarse-graining procedure and can be carried out on a time series as follows. For a given time series $\{a_1, a_2, \dots, a_n\}$, where $a_i = F_x(i)$, multiple coarse-grained time series are constructed by averaging the data points within nonoverlapping windows of increasing length as shown in Fig. 7. Each element of

Fig. 6 Recurrence rate (RR) (a) and determinism (DET) (b) versus the threshold value ϵ . Note, the peaks for $\epsilon \approx 0.4$. The highest peak corresponds to the speed $\omega = 8,000$ rpm

Fig. 7 The schematic picture of multiscaled grid approximation applied to the time series $\{a_1, a_2, \dots, a_n\}$, where $a_i = F_x(i)$ to estimate the multiscaled entropy

the coarse-grained time series is computed according to the equation:

$$a_j^{(r)} = \frac{1}{r} \sum_{i=(j-1)r+1}^{jr} a_i, \quad (9)$$

where r represents the scale factor, and $1 \leq j \leq N/r$, the length of each coarse-grained time series is equal to N/r . Note that for $r = 1$, the coarse-grained time series is simply the original time series. The sample entropy S_E , proposed originally by Richman and Moorman [31], is calculated for the sequence of m consecutive data points which are ‘similar’ to each other and will remain similar when one or more consecutive data points are included [31, 32]. Here, “similar” means that the value of a specific measure of distance is less than a prescribed amount h . Therefore, the sample entropy depends on two parameters r and h . Thus, we have

$$S_E(r, h, N) = \ln \frac{\sum_{i=1}^{N/r} \tilde{a}(i, r, h)}{\sum_{i=1}^{N/(r+1)} \tilde{a}(i, r+1, h)}, \quad (10)$$

where $\tilde{a}(i, r, h)$ denotes the group of normalized $a(i)$ fulfilling the similarity criterion of h . The sample en-

tropy has been calculated for each of the coarse-grained time series of F_x and then plotted as a function of the scale factor r and the similarity h . The results for speeds $\omega = 2,000$ – $8,000$ rpm (Fig. 2a–e) are shown in Fig. 8a–e. Examining these figures, one can see that Fig. 8b shows nonuniform transition of higher to lower S_E showing the increase in the small r limit and fairly small h whereas Fig. 8d, e mimic the similar tendency in larger r and higher h . Namely, in Fig. 8e one can see the waving effect. These opposite tendencies are related with some additional noticeable modulation in high (Fig. 8b) and small (Fig. 8d, e) frequency limits. Looking more carefully into these figures one can notice that for very small h , Fig. 8d resembles Fig. 8b. In between that cases, Fig. 8c ($\omega = 3,500$ rpm) shows a lowering tendency in S_E indicating some change in dynamics (possibly a bifurcation point). Apart from that one can notice some weak modulations in Fig. 8a. This could be also related to smaller frequency modulation as Fig. 8d, e. The dynamical response of the higher spindle velocity is more complex showing the importance of multiple time scales. In fact, basing on the peculiar dependence of DET (Fig. 6b), the similar conclusions can be drawn from recurrence analysis.

Fig. 8 Sample entropy SE calculated for normalized time series versus the scaling factor r and the similarity factor h . Panels a–e in this figure correspond to panels a–e, respectively in Fig. 2

4 Conclusions and last remarks

By applying RP and RQA methods to the time series obtained with increasing speed of milling we have observed a non monotonous increasing tendency in the parameter DET. It is showing a clear peak for the lower range of ϵ . This may be connected to the two opposing effects appearing in the system. Firstly, the initial increase is related to obvious decreasing of rotation period for increasing speed. One should remember that we have used the same fixed embedding for all cases. On the one hand, it enables to make results comparable. On the other, especially for high speed the embedding is not relevant for short neighbor recurrences and this is resulting in higher density of lines along the diagonal.

For higher speeds, a new time scale of system response in higher frequency region emerges. As the amplitudes are similar this effect can work as a type of modulation. The above comments can be supported by the corresponding power spectra presented in Fig. 9. Note that the first peaks, denoted by “1” in each of Fig. 9a–e, correspond to rotational speed of the spindle and they are growing with the increasing speed. The other peaks numbered as “3” and “4” appear for localized frequencies in the higher frequency region. In the case of the peak frequency “2”, one can see some difference in location for $\omega = 8,000$ rpm comparing to the fairly fixed value in lower spindle speeds. Comparing the highs of the corresponding peaks: “1”, “2”, and

“3” (in all the considered cases a–e), it is clear that the dynamical response ‘d’ and ‘e’ is highly nonlinear. Note that the higher harmonics (“2” and “3”) are more pronounced and form the series of spindle frequency multiples.

It should be also noted that all additional higher frequencies lead to some additional modulation of the measured force component. Consequently, the multi-scaled entropy showed the nonhomogenous behavior of S_E against the scaling and similarity factors and confirmed the main changes in short period (intermittences) as well as long-period (modulation) changes.

In summary, we would like to stress that then going through the analysis of recurrences we noticed that the RQA parameters could be used to improve diagnostics and invent a chatter feedback-control procedure in the milling process [1]. Especially, DET can be used in the spindle speed control to eliminate the vibrations of higher harmonics. To generalize the conclusions of our results we are preparing now new series of experiments for different workpiece materials including metals and composites reinforced by glass fibers. The new results will be reported in a separate publication.

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Fig. 9 Power spectrum for different speed values ($\omega = 2,000$, 3,500, 5,000, 6,500, and 8,000 rpm). First peaks correspond to rotational speed of the spindle ($f = \omega/60 = 33.3, 58.3, 83.3, 108.3$, and 133.3 Hz, respectively). Note that the $P(f)$ is in the dimensionless units as time series shown in Fig. 2 have been normalized by σ_{Fx} specified in Table 1

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