ERRATUM

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Erratum to: Discrete structures equivalent to nonlinear Dirichlet and wave equations

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1 A perturbed wave equation amenable to exact reduction

In [2] we have considered the problem of finding T -periodic solutions $u(t, x) \in H := H_0^1([0, T] \times \mathbb{T}^n, \mathbb{R})$ of the nonlinear wave equation

$$\begin{cases} \square u(t, x) = F(u), & t \in \mathbb{R}, x \in \mathbb{T}^n, \\ u(t, x) = u(t + T, x), \end{cases} \quad (1)$$

where $\square = \frac{\partial^2}{\partial t^2} - \frac{\partial^2}{\partial x^2}$ is the D'Alembert wave operator while F is a Nemitski Lipschitz operator. To produce an exact finite reduction in the wave equation we needed the inverse operator of the D'Alembertian. To arrive to this scope we first expanded $v \in H$ in Fourier series and then we laid down the formal explicit component-wise inverse of $\square$:

$$\square^{-1}(v_k(t)) := \left(-\frac{1}{\sin(|k|T)} \int_0^T \sin(|k|(T-s)) v_k(s) ds \right) \frac{\sin(|k|t)}{|k|} + \int_0^t \frac{\sin(|k|(t-s))}{|k|} v_k(s) ds, \quad (2)$$

where $k \in \mathbb{Z}^n$, $k \neq 0$.

However, this formula is not valid in general, i.e., the D'Alembert operator $\square$ does not admit a bounded inverse in H both with rational and irrational periods T . This could be overcome by considering numbers of constant type

$$|Tk - l| > Ck^{-1}, \quad (3)$$

as in [4]; this, however, is not compatible with the contractivity needed for the reduction procedure. We have instead adopted the standard approach by Rabinowitz [5], i.e., we considered the vibrating string ($n = 1$), we

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took T rational multiple of π , and we restricted the domain of $\square$ to the orthogonal complement of the kernel of $\square$. The restricted problem appears in [1,3] among others. Adopting this format the kernel of the D'Alembertian becomes trivial and the existence of a bounded inverse $\square^{-1}$ is assured. In this way it is possible for instance to relax monotonicity conditions on F and obtain existence results as well [3]. The procedure described in [2] is still valid and leads to the correct conclusions. A derivation of these results in the hypotheses discussed here has been exposed in full details in our subsequent work [6].

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