ERRATUM

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Erratum to: On the proper formulation of Maxwellian electrodynamics for continuum mechanics

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In the original publication of the article, Maxwell's equations for continuum mechanics contain a few errors of sign and indicial notation. These corrections do little to alter the conclusions of the paper except that the resultant equations are more symmetric.

Equation 8 has a small typographical error and should be

$$\tilde{g}_{ik} \doteq \tilde{g}_i \tilde{g}_k - \tilde{g}_{ik}. \quad (1)$$

Equation 17(b) has a slightly more significant mistake in its treatment of the metric element, and should read

$$\tilde{m}^{ik} = -\frac{\epsilon^{ik\tilde{\ell}} \tilde{h}_{\tilde{\ell}}}{\sqrt{-\tilde{g}}}. \quad (2)$$

In Eq. 19, the order of $\tilde{\alpha}$ and $\tilde{\beta}$ are reversed, and the argument should be $m^{\tilde{\beta}\tilde{\alpha}}$. In Eq. 21, there is a spurious minus sign in $\sqrt{\tilde{g}}$. The corrected expression is

$$\frac{1}{\sqrt{\tilde{g}}} \left[\epsilon^{ik\tilde{\ell}} \frac{\partial \tilde{h}_{\tilde{\ell}}}{\partial \tilde{x}^k} - \frac{\partial}{\partial \tilde{t}} \left(\sqrt{\tilde{g}} \tilde{d}^i \right) \right] = \tilde{j}^i. \quad (3)$$

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In Eq. 22, tildes are missing on some of the α and β indices. The order should also be reversed to agree with the revised Eq. 19. A tilde on α is missing on $\tilde{j}^\alpha$ in the line after Eq. 22. A tilde is missing, and a spurious factor of c appears in Eq. 30. The corrected version is

$$\tilde{v}^i \doteq c \frac{d\tilde{x}^i}{d\tilde{x}^0} = -\frac{\partial \tilde{x}^i}{\partial x^i} v^i. \quad (4)$$

A minus sign is missing in the $\tilde{g}_{\tilde{i}\tilde{k}}$ term in Eq. 34. The equation should be

$$\tilde{g}_{\tilde{i}\tilde{k}} = -\delta_{ik} \frac{\partial x^i}{\partial \tilde{x}^i} \frac{\partial x^k}{\partial \tilde{x}^k}. \quad (5)$$

Unlike the previous errors, this one carries into the remainder of the paper, causing errors in later expressions. The right Cauchy–Green tensor C_{IK} expression, Eq. 35, should then be

$$\tilde{g}_{\tilde{i}\tilde{k}} = -\delta_{ik} \frac{\partial x^i}{\partial \tilde{x}^i} \frac{\partial x^k}{\partial \tilde{x}^k} = -\delta_i^I \delta_k^K \delta_{ik} \frac{\partial x^i}{\partial X^I} \frac{\partial x^k}{\partial X^K} = -\delta_i^I \delta_k^K C_{IK}, \quad (6)$$

and Eq. 36 is therefore

$$\tilde{g}^{00} = 1, \quad \tilde{g}^{\tilde{i}0} = \tilde{g}^{0\tilde{i}} = \frac{\tilde{v}^i}{c}, \quad \tilde{g}^{\tilde{i}\tilde{k}} = -\delta^{ik} \frac{\partial \tilde{x}^i}{\partial x^i} \frac{\partial \tilde{x}^k}{\partial x^k}. \quad (7)$$

In Eq. 37, the derivatives are with respect to the four-vector $\tilde{x}^\alpha$, not the purely spatial components $\tilde{x}^i$ and should therefore be

$$\tilde{n}_{\alpha\beta} = \frac{\partial x^\alpha}{\partial \tilde{x}^\alpha} \frac{\partial x^\beta}{\partial \tilde{x}^\beta} n_{\alpha\beta}. \quad (8)$$

Equation 52 is obviously incorrect as the units do not match. It should read

$$\tilde{x}^0 = \tilde{a}x^0 + \tilde{s}_i \tilde{x}^i. \quad (9)$$

A sign error appears in Eq. 54, and is carried to other equations. The corrected expressions are

$$\frac{\partial x^0}{\partial \tilde{x}^0} = \frac{1}{\tilde{a}}, \quad \frac{\partial x^0}{\partial \tilde{x}^i} = -\frac{\tilde{s}_i}{\tilde{a}}, \quad \frac{\partial x^i}{\partial \tilde{x}^0} = \frac{1}{\tilde{a}} \frac{v^i}{c}, \quad \frac{\partial x^i}{\partial \tilde{x}^i} = \delta_i^I \frac{\partial x^i}{\partial X^I} - \frac{\tilde{s}_i}{\tilde{a}} \frac{v^i}{c}. \quad (10)$$

Incorporating these changes and correcting for an erroneous square root, Eq. 55 should read

$$\tilde{s}_i = -\frac{\tilde{a} \delta_{ij} \frac{\partial x^j}{\partial \tilde{x}^i} \frac{v^i}{c}}{1 - \delta_{k\ell} \frac{v^k v^\ell}{c^2}}. \quad (11)$$

The sign error propagates further to Eq. 58 which should now be

$$\frac{\partial \tilde{x}^0}{\partial x^0} = 1, \quad \frac{\partial \tilde{x}^0}{\partial x^i} = -\delta_{ij} \frac{v^j}{c}, \quad \frac{\partial \tilde{x}^i}{\partial x^0} = \frac{\tilde{v}^i}{c}, \quad \frac{\partial \tilde{x}^i}{\partial x^i} = \frac{\partial \tilde{x}^i}{\partial x^i}. \quad (12)$$

In Eq. 60, the $\tilde{g}_{\tilde{i}\tilde{j}}$ term should be

$$\tilde{g}_{\tilde{i}\tilde{j}} = -\tilde{g}_{\tilde{i}\tilde{j}} = -\delta_{ij} \frac{\partial x^i}{\partial \tilde{x}^i} \frac{\partial x^j}{\partial \tilde{x}^j}, \quad (13)$$

as the expression presented in the paper is obviously not a proper indicial expression. In Eq. 61, the indices on $\tilde{g}^{\tilde{i}\tilde{j}}$ should be raised. The sign corrections lead to changes in the definitions given by Eqs. 62–64. The corrected definitions are

$$\tilde{e}^{\tilde{i}} \doteq \tilde{g}^{\tilde{i}\tilde{j}} \tilde{e}_{\tilde{j}}, \quad \tilde{d}_{\tilde{i}} \doteq \tilde{g}_{\tilde{i}\tilde{j}} \tilde{d}^{\tilde{j}}, \quad (14)$$

$$\tilde{h}^{\tilde{i}} \doteq \tilde{g}^{\tilde{i}\tilde{j}} \tilde{h}_{\tilde{j}}, \quad \tilde{b}_{\tilde{i}} \doteq \tilde{g}_{\tilde{i}\tilde{j}} \tilde{b}^{\tilde{j}}, \quad (15)$$

$$\tilde{p}_{\tilde{i}} \doteq \tilde{g}_{\tilde{i}\tilde{j}} \tilde{p}^{\tilde{j}}, \quad \tilde{m}^{\tilde{i}} \doteq \tilde{g}^{\tilde{i}\tilde{j}} \tilde{m}_{\tilde{j}}. \quad (16)$$

Equations 66–67 now become

$$\tilde{m}_{\tilde{i}0} = -c\tilde{d}_{\tilde{i}}, \quad \tilde{m}_{\tilde{i}\tilde{j}} = -\sqrt{\tilde{g}}\epsilon_{\tilde{i}\tilde{j}\tilde{k}}\tilde{h}^{\tilde{k}}, \quad (17)$$

$$\tilde{p}_{\tilde{i}0} = c\tilde{p}_{\tilde{i}}, \quad \tilde{p}_{\tilde{i}\tilde{j}} = -\sqrt{\tilde{g}}\epsilon_{\tilde{i}\tilde{j}\tilde{k}}\tilde{m}^{\tilde{k}}. \quad (18)$$

The sign error then propagates to Eqs. 68–73, which become

$$\tilde{e}_{\tilde{i}} = \frac{\partial x^i}{\partial \tilde{x}^{\tilde{i}}} \left(e_i + \epsilon_{ijk} v^j b^k \right), \quad \tilde{b}^{\tilde{i}} = \frac{\partial \tilde{x}^{\tilde{i}}}{\partial x^i} \left(b^i - \epsilon^{ijk} \frac{v_j e_k}{c^2} \right), \quad (19)$$

$$\tilde{e}^{\tilde{i}} = \frac{\partial \tilde{x}^{\tilde{i}}}{\partial x^i} \left(e^i + \epsilon^{ijk} v_j b_k \right), \quad \tilde{b}_{\tilde{i}} = \frac{\partial x^i}{\partial \tilde{x}^{\tilde{i}}} \left(b_i - \epsilon_{ijk} \frac{v^j e^k}{c^2} \right), \quad (20)$$

$$\tilde{d}^{\tilde{i}} = \frac{\partial \tilde{x}^{\tilde{i}}}{\partial x^i} \left(d^i + \epsilon^{ijk} \frac{v_j h_k}{c^2} \right), \quad \tilde{h}_{\tilde{i}} = \frac{\partial x^i}{\partial \tilde{x}^{\tilde{i}}} \left(h_i - \epsilon_{ijk} v^j d^k \right), \quad (21)$$

$$\tilde{p}^{\tilde{i}} = \frac{\partial \tilde{x}^{\tilde{i}}}{\partial x^i} \left(p^i - \epsilon^{ijk} \frac{v_j m_k}{c^2} \right), \quad \tilde{m}_{\tilde{i}} = \frac{\partial x^i}{\partial \tilde{x}^{\tilde{i}}} \left(m_i + \epsilon_{ijk} v^j p^k \right), \quad (22)$$

$$p^i = \frac{\partial x^i}{\partial \tilde{x}^{\tilde{i}}} \left(\tilde{p}^{\tilde{i}} - \frac{\epsilon^{\tilde{i}\tilde{j}\tilde{k}}}{\sqrt{\tilde{g}}} \tilde{v}_{\tilde{j}} \tilde{m}_{\tilde{k}} \right), \quad m_i = \frac{\partial \tilde{x}^{\tilde{i}}}{\partial x^i} \left(\tilde{m}_{\tilde{i}} + \sqrt{\tilde{g}} \epsilon_{\tilde{i}\tilde{j}\tilde{k}} \tilde{v}^{\tilde{j}} \tilde{p}^{\tilde{k}} \right), \quad (23)$$

$$\tilde{\rho} = \rho - \frac{v_i j^i}{c^2}, \quad \tilde{j}^{\tilde{i}} = \frac{\partial \tilde{x}^{\tilde{i}}}{\partial x^i} \left(j^i - \rho v^i \right). \quad (24)$$

Finally, the corrected versions of Tables 1, 2 and 3 are shown. As can be seen, after these corrections for sign errors, the time orthogonal transformation equations exhibit a completely symmetric relationship. They are also in complete agreement with the well-known frame transformation laws for electromagnetic field quantities in the limit of rigid motion. In contrast, even with the corrections, the transformation rules commonly used in continuum mechanics [1–4] remain asymmetric. As mentioned in the paper, this asymmetry is a source of confusion and possible errors in formulations that rely on them.

Table 1 Transformation of the elements of the electromagnetic tensor in different formulations

	e	b
Universal time, $\tilde{n}_{\alpha\beta}$	$\tilde{e}_{\tilde{i}} = \frac{\partial x^i}{\partial \tilde{x}^{\tilde{i}}} (e_i + \epsilon_{ijk} v^j b^k)$	$\tilde{b}^{\tilde{i}} = \frac{\partial \tilde{x}^{\tilde{i}}}{\partial x^i} b^i$
Universal time, $\tilde{n}^{\alpha\beta}$	$\tilde{e}^{\tilde{i}} = \frac{\partial \tilde{x}^{\tilde{i}}}{\partial x^i} e^i$	$\tilde{b}_{\tilde{i}} = \frac{\partial x^i}{\partial \tilde{x}^{\tilde{i}}} \left(b_i - \epsilon_{ijk} \frac{v^j e^k}{c^2} \right)$
Time orthogonal, $\tilde{n}_{\alpha\beta}$	$\tilde{e}_{\tilde{i}} = \frac{\partial x^i}{\partial \tilde{x}^{\tilde{i}}} (e_i + \epsilon_{ijk} v^j b^k)$	$\tilde{b}^{\tilde{i}} = \frac{\partial \tilde{x}^{\tilde{i}}}{\partial x^i} \left(b^i - \epsilon^{ijk} \frac{v_j e_k}{c^2} \right)$
Lax and Nelson [3]	$E_I = \frac{\partial x_i}{\partial \tilde{x}_I} (e_i + \epsilon_{ijk} v^j b_k)$	$B_I = \sqrt{C} \frac{\partial X_I}{\partial x_i} b_i$
Yang and Batra [4]	$E_I = \sqrt{C} \frac{\partial X_I}{\partial x_i} e_i$	(Electrostatic)
Dorfmann and Ogden [2]	$E_I = \frac{\partial x_i}{\partial \tilde{x}_I} e_i$	(Electrostatic)
Clayton [1]	$E_I = \frac{\partial x_i}{\partial \tilde{x}_I} e_i$	(Electrostatic)

Table 2 Transformation of the elements of the displacement tensor in different formulations

	d	h
Universal time, $\tilde{m}^{\alpha\beta}$	$\tilde{d}^{\tilde{i}} = \frac{\partial \tilde{x}^{\tilde{i}}}{\partial x^i} d^i$	$\tilde{h}_{\tilde{i}} = \frac{\partial x^i}{\partial \tilde{x}^{\tilde{i}}} (h_i - \epsilon_{ijk} v^j d^k)$
Universal time, $\tilde{m}_{\alpha\beta}$	$\tilde{d}_{\tilde{i}} = \frac{\partial x^i}{\partial \tilde{x}^{\tilde{i}}} \left(d_i + \epsilon_{ijk} \frac{v^j h^k}{c^2} \right)$	$\tilde{h}^{\tilde{i}} = \frac{\partial \tilde{x}^{\tilde{i}}}{\partial x^i} h^i$
Time orthogonal, $\tilde{m}^{\alpha\beta}$	$\tilde{d}^{\tilde{i}} = \frac{\partial \tilde{x}^{\tilde{i}}}{\partial x^i} \left(d^i + \epsilon^{ijk} \frac{v_j h_k}{c^2} \right)$	$\tilde{h}_{\tilde{i}} = \frac{\partial x^i}{\partial \tilde{x}^{\tilde{i}}} (h_i - \epsilon_{ijk} v^j d^k)$
Lax and Nelson [3]	$D_I = \sqrt{C} \frac{\partial X_I}{\partial x_j} d_i$	$H_I = \frac{\partial x_i}{\partial \tilde{x}_I} (h_i - \epsilon_{ijk} v_j d_k)$
Yang and Batra [4]	$D_I = \sqrt{C} \frac{\partial X_I}{\partial x_j} d_i$	(Electrostatic)
Dorfmann and Ogden [2]	$D_I = \sqrt{C} \frac{\partial X_I}{\partial x_j} d_i$	(Electrostatic)
Clayton [1]	$D_I = \sqrt{C} \frac{\partial X_I}{\partial x_j} d_i$	(Electrostatic)

Table 3 Transformation of the elements of the polarization tensor in different formulations

	p	m
Universal time, $\tilde{p}^{\alpha\beta}$	$\tilde{p}^{\tilde{i}} = \frac{\partial \tilde{x}^{\tilde{i}}}{\partial x^i} p^i$	$\tilde{m}_{\tilde{i}} = \frac{\partial x^i}{\partial \tilde{x}^{\tilde{i}}} (m_i + \epsilon_{ijk} v^j p^k)$
Universal time, $\tilde{p}_{\alpha\beta}$	$\tilde{p}_{\tilde{i}} = \frac{\partial x^i}{\partial \tilde{x}^{\tilde{i}}} \left(p_i - \epsilon_{ijk} \frac{v^j m^k}{c^2} \right)$	$\tilde{m}^{\tilde{i}} = \frac{\partial \tilde{x}^{\tilde{i}}}{\partial x^i} m^i$
Time orthogonal, $\tilde{p}^{\alpha\beta}$	$\tilde{p}^{\tilde{i}} = \frac{\partial \tilde{x}^{\tilde{i}}}{\partial x^i} \left(p^i - \epsilon^{ijk} \frac{v_j m_k}{c^2} \right)$	$\tilde{m}_{\tilde{i}} = \frac{\partial x^i}{\partial \tilde{x}^{\tilde{i}}} (m_i + \epsilon_{ijk} v^j p^k)$
Lax and Nelson [3]	$P_I = \sqrt{C} \frac{\partial X_I}{\partial x_j} p_i$	$M_I = \frac{\partial x^i}{\partial X^I} m_i$
Yang and Batra [4]	$P_I = \sqrt{C} \frac{\partial X_I}{\partial x_j} p_i$	(Electrostatic)
Dorfmann and Ogden [2]	$P_I = \sqrt{C} \frac{\partial X_I}{\partial x_i} p_i$	(Electrostatic)
Clayton [1]	$P_I = \frac{\partial x^i}{\partial X^I} p_i$	(Electrostatic)

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