

ERRATUM TO: AN EQUIVALENCE OF FUSION CATEGORIES

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1. The author is grateful to Yi-Zhi Huang who has found the following gap in the proof of Proposition 3.6 and hence in the proof of the main Theorems 3.4, 4.3: In the second and third lines before the end of proof of Proposition 3.6 (page 265), it is stated (and proved) that one vector is proportional to another one. However, the coefficient of proportionality may vanish, hence in the notations of Proposition 3.6 it is only proved that $L = H'' \subset H' = H$.

In order to prove that $L = H'' = H' = H$ as stated in Proposition 3.6, we argue as follows:

Theorem 4.4 is proved e.g. in [Fal94, Tel95]. It means that the K -rings of monoidal categories $\tilde{\mathcal{O}}_{\pm\kappa}$ are based isomorphic. It means that $\dim L = \dim H$, and hence the above inclusion $L \subset H$ must be an isomorphism.

The proof of Proposition 3.6 (and hence Theorems 3.4, 4.3) is now complete. Note however that Theorem 4.4 is not deduced from the main Theorem 4.3 but instead is used in its proof.

2. The following minor corrections are also due to Yi-Zhi Huang:
 - (a) Page 249, line 6 of the first paragraph of Introduction: replace “It is known” by “It is conjectured”, so that the sentence reads: “It is conjectured (see e.g. [MoSe] or [BFM]) that $\tilde{\mathcal{O}}_k$ has the structure of a rigid braided tensor category.”
 - (b) Page 261, line 2: replace $\mathbf{Hom}_{\tilde{\mathcal{O}}_{\pm\kappa}}(V, W) \times \mathbf{Hom}_{\tilde{\mathcal{O}}_{\pm\kappa}}(W, V) \rightarrow \mathbb{C}$ by $\mathbf{Hom}_{\tilde{\mathcal{O}}_{-\kappa}}(V, W) \times \mathbf{Hom}_{\tilde{\mathcal{O}}_{-\kappa}}(W, V) \rightarrow \mathbb{C}$.

This pairing arises from rigidity of $\tilde{\mathcal{O}}_{-\kappa}$, and the corresponding pairing at the positive level would arise from rigidity of $\tilde{\mathcal{O}}_{\kappa}$ which is not yet established in Section 3, and is only proved in Section 4 as a result of tensor equivalence with

$\tilde{\mathcal{O}}_{-\kappa}^{opp}$. This proof of rigidity of $\tilde{\mathcal{O}}_{\kappa}$ does not work in the cases of E_6 level 1, E_7 level 1, and E_8 levels 1 and 2 (see the first paragraph of Section 2.6).

(c) Page 261, line -4 : replace j_{1*} by \mathfrak{D} .

(d) Page 265, Section 4: replace any occurrence of $\tilde{\mathcal{O}}_{-\kappa}$ by $\tilde{\mathcal{O}}_{-\kappa}^{opp}$.

(e) Page 266, add the following between lines 3 and 4:

We denote by $G : \tilde{\mathcal{O}}_{\kappa} \rightarrow \tilde{\mathcal{O}}_{-\kappa}$ the composition of $F : \tilde{\mathcal{O}}_{\kappa} \rightarrow \tilde{\mathcal{O}}_{-\kappa}^{opp}$ with the rigidity $\tilde{\mathcal{O}}_{-\kappa}^{opp} \rightarrow \tilde{\mathcal{O}}_{-\kappa}$. Then ψ still gives the same named morphism of functors $\psi : G \circ \tilde{\otimes} \xrightarrow{\sim} \tilde{\otimes} \circ (G \times G)$.

(f) Page 266, replace Theorem 4.3 by:

The tensor functor $(G, \psi) : (\tilde{\mathcal{O}}_{\kappa}, \tilde{\otimes}) \rightarrow (\tilde{\mathcal{O}}_{-\kappa}, \tilde{\otimes})$ is a tensor equivalence for $\kappa \geq \check{h} + 3$.

3. A calculation of $K(\tilde{\mathcal{O}}_{\kappa})$ using only the representation theory of vertex operator algebras (and not using algebraic geometry or loop groups) was given later in [Hua08a].

A proof of rigidity of $\tilde{\mathcal{O}}_{\kappa}$ including in particular the cases of E_6 level 1, E_7 level 1, and E_8 levels 1 and 2, was given in [Hua08b].

References

- [Fal94] G. FALTINGS. A proof for the Verlinde formula. *Journal of Algebraic Geometry*, (2)**3** (1994), 347–374.
- [Tel95] C. TELEMAN. Lie algebra cohomology and the fusion rules. *Communications in Mathematical Physics*, (2)**173** (1995), 265–311.
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- [Hua08b] Y.-Z. HUANG. Rigidity and modularity of vertex tensor categories. *Communications in Contemporary Mathematics*, **10** (2008), 871–911.

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