ERRATUM TO: ON REPRESENTATIONS OF INTEGERS IN THIN SUBGROUPS OF $SL_2(\mathbb{Z})$

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Let $\Gamma < SL(2, \mathbb{Z})$ be finitely generated, free, with no parabolic elements, and having critical exponent $1/2 < \delta < 1$. As such, there exists a finite symmetric set $\mathcal{A} := \{A_1, A_1^{-1}, A_2, A_2^{-1}, \dots, A_k, A_k^{-1}\}$ of generators with no relations, so that every $\gamma \in \Gamma$ is expressed uniquely as some reduced word $\gamma = B_1 B_2 \cdots B_m$ with $B_j \in \mathcal{A}$. (Reduced means no annihilations, $B_j B_{j+1} \neq I$). Mimicing [BK, §3, (3.2)], we construct the following exponential sum.

Fix $N \gg 1$ and $0 < \sigma < 1/4$. Let Ξ be a subset of Γ containing elements of Frobenius norm at most $N^{1/2}$, such that all elements $\xi \in \Xi$, when written as a reduced word in the generators of Γ , start with the same letter.

Similarly, let Π be a subset of Γ containing elements of Frobenius norm at most $N^{1/2-\sigma}$, and all elements $\varpi \in \Pi$, when written as a reduced word in the generators of Γ , ending in the same letter (ensuring that it is not the inverse of the letter which starts all elements of Ξ).

Then for $\theta \in [0, 1]$, and primitive $v_0, w_0 \in \mathbb{Z}^2$, let

$$S_N(\theta) := \sum_{\xi \in \Xi} \sum_{\varpi \in \Pi} \sum_{\substack{\gamma \in \Gamma \\ \|\gamma\| < N^\sigma}} e(\langle v_0 \cdot \gamma \varpi \xi, w_0 \rangle \theta). \quad (3.2')$$

(We have already changed the definitions of Ξ , Π and S_N to fix a minor notational inconsistency: in [BK, §3-4], every appearance of “ $\xi \varpi$ ” should be replaced by “ $\varpi \xi$ ”, and similarly “ ${}^t \varpi {}^t \xi$ ” by “ ${}^t \xi {}^t \varpi$ ”. The latter appearances are correct.)

The purpose of this construction was twofold. First, by the pigeonhole principle, Ξ and Π can be chosen so that they are “as large as they should be,” that is, $|\Xi| \gg N^\delta$ and $|\Pi| \gg N^{\delta(1-2\sigma)}$. Second, and most importantly, the concatenation $\Pi \cdot \Xi$ should be unique, that is,

$$\text{if } \varpi \xi = \varpi' \xi', \text{ with } \varpi, \varpi' \in \Pi \text{ and } \xi, \xi' \in \Xi, \text{ then } \varpi = \varpi' \text{ and } \xi = \xi'. \quad (*)$$

By forcing the ending letter of ϖ to differ from the inverse of the starting letter of ξ , we have ensured that there is no annihilation in the concatenation $\varpi \cdot \xi$.

Unfortunately, this does not guarantee (*). Indeed if $A, B \in \Gamma$ are free, then the concatenation of $\Pi = \{A, ABA\}$ with $\Xi = \{B, BAB\}$ contains both $A \cdot BAB$ and $ABA \cdot B$.

On the other hand, (*) *would* be guaranteed if we restricted Ξ , say, to contain elements which, in addition to having the same starting letter, were also all of the same length.

So we amend the construction as follows. For a geometrically finite group with no parabolics, the wordlength metric $\ell(\cdot)$ is related to Frobenius norm $\|\cdot\|$ by [F, Lem., p. 213]:

$$\log \|\gamma\| \ll \ell(\gamma) \ll \log \|\gamma\|,$$

with implied constants depending only on Γ . Hence in the ball in Γ of elements having Frobenius norm at most $N^{1/2}$, the wordlengths $\ell(\gamma)$ are dominated by a constant multiple of $\log N$. Then by the pigeonhole principle, there is a subset $\tilde{\Xi}$ consisting of words of the same wordlength, having size

$$|\tilde{\Xi}| \gg N^\delta / \log N.$$

We now proceed as before, selecting from within this set a subset Ξ consisting of elements with the same starting letter. The rest of the argument works in the same way, save a log loss in the major arcs estimate, where Theorem 4.1 is replaced by the estimate

$$\mathcal{M}_N(n) \gg \frac{1}{\log \log(|n| + 10)} \frac{N^{2\delta-1}}{\log N},$$

if n is admissible and not in the exceptional set.

Fortunately for us, such a loss is inconsequential, due to the power savings on the minor arcs (Theorem 8.18). The proof of Theorem 8.19 now bounds the size of the exceptional set by

$$|\mathfrak{E}(N)| \ll N^{1-\eta} (\log N)^2 (\log \log N)^2,$$

which is still plenty for our purposes.

References

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