ERRATA FOR: “ON RANDOMIZED ONE-ROUND COMMUNICATION COMPLEXITY”

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In the paper “On randomized one-round communication complexity” (Kremer *et al.* 1999), one of the results we claimed was:

THEOREM 5.1. $R^{\parallel, \text{pub}}(f) = \Theta(R^{A \rightarrow B, \text{pub}}(f) + R^{B \rightarrow A, \text{pub}}(f)).$

Here $R^{\parallel, \text{pub}}(f)$ denotes the simultaneous public-coin communication complexity of f , that is, when both parties send a message to a “referee” who computes the output without seeing any part of the input. The notation $R^{A \rightarrow B, \text{pub}}(f)$ (resp. $R^{B \rightarrow A, \text{pub}}(f)$) stands for the one-way public-coin communication complexity of f when the message is sent from party A to party B (resp. from party B to party A).

It is clear that $R^{\parallel, \text{pub}}(f) = \Omega(R^{A \rightarrow B, \text{pub}}(f) + R^{B \rightarrow A, \text{pub}}(f))$, since both party A and party B can simulate the referee’s side of the protocol. However, the main argument in our proof of the upper bound $R^{\parallel, \text{pub}}(f) = O(R^{A \rightarrow B, \text{pub}}(f) + R^{B \rightarrow A, \text{pub}}(f))$ had a flaw: we claimed that for every probability distribution μ on $X \times Y$, $D^{\parallel, \mu}(f) = O(D^{A \rightarrow B, \mu}(f) + D^{B \rightarrow A, \mu}(f))$, where $D^{\parallel, \mu}(f)$ is the simultaneous μ -distributional complexity of f , and $D^{A \rightarrow B, \mu}(f)$ and $D^{B \rightarrow A, \mu}(f)$ are the one-way μ -distributional complexities. It turns out that this claim is *not* true for every distribution, but only for rectangular (product) distributions on $X \times Y$.

This left the possibility that the theorem is still correct, but that a different argument is required. However, recently, Bar-Yosef *et al.* (2002) have been able to show that there exists a function f for which $R^{A \rightarrow B, \text{pub}}(f) = O(\log n)$ and $R^{B \rightarrow A, \text{pub}}(f) = O(\log n)$ while $R^{\parallel, \text{pub}}(f) = \Omega(\sqrt{n})$, contradicting our (evidently, false) theorem.

As a consequence we also need to retract our claim that $R^{\parallel, \text{pub}}(\text{INP}_{2,2}) = O(1)$, where $\text{INP}_{2,2}$ is the inner product of pairs of vectors whose L_2 norm equals 1, and to modify Proposition 4.5 from stating that $R^{\parallel}(\text{INP}_{2,2}) = \Theta(\sqrt{n})$, to stating that $R^{\parallel}(\text{INP}_{2,2}) = \Omega(\sqrt{n})$.

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References

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