Book Review

Benoit Mandelbrot. A Life in Many Dimensions by Michael Frame and Nathan Cohen (eds.), World Scientific, 2015; ISBN: 978-981-4366-06-9

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Mathematics played a dominant role in our understanding of natural and physical sciences. In 1960 E. Wigner said: “The miracle of the appropriateness of the language of mathematics for the formulation of the laws of physics is a wonderful gift which we neither understand nor deserve” [Comm. Pure Appl. Math., vol. 13, no. I (1960)]. This refers also to the fractal geometry.

This book commemorates Prof. Benoit Mandelbrot, who passed away on 14 October 2010 at the age of 85. Prof. Mandelbrot was undoubtedly one of the most significant mathematicians in our century.

B. Mandelbrot’s most widely recognized contributions were the mathematics for finance and the fractal concept and its consequences. But I think that Mandelbrot’s greatest achievement was that he revolutionized the approach that researchers view subjects and phenomena, both in Mathematics and in Natural Sciences.

The book contains 26 articles by as many authors on different aspects of the fractal geometry and fractal analysis. The diversity of subject matter should not be surprising since the wide range of scientific problems involving fractal objects includes fields as diverse as statistical physics, engineering, computer science, economic and finance, social science, music, painting, and of course, mathematics.

After the Prefaces, dedicated to B. Mandelbrot, a short chapter 1 contains a personal reminiscence by Aliette Mandelbrot.

Chapter 2 by M. Frame is devoted to a brief discussion of some aspects of the fractal analysis. The concepts of scaling, deterministic and random fractals, similarity dimension, fractal sets, multifractals, etc. are introduced and compactly explained. Some personal reflections on certain aspects of B. Mandelbrot’s work are also included.

Chapter 3 by P. Abry, S. Jaffard and H. Wendt focuses on the basic notions of the fractal geometry, namely, scale invariance, self-similarity and non-integer dimension. The concept of self-similarity is explained in the context of the celebrated Weierstrass–Mandelbrot function. It is defined on the interval $[0, 1]$, which although continuous everywhere is differentiable nowhere. The notion of statistical self-similarity is introduced and discussed also. The important fact is that those notions are closely related to multifractals and wavelets. The multifractal formalism and the wavelet techniques are the theme of the next sections. Subsequent section is the heart of this chapter and is dedicated to real-world applications of fractal analysis to the study of various types of real phenomena, for example the hydrodynamic turbulence, finance time series, fetal heart rate fluctuations, Internet traffic modelling, etc. The last section addresses the relevance and the future developments of the multifractal analysis. In particular, the multifractal toolbox written in Matlab is mentioned in this context.

Chapter 4 by M. F. Barnsley and B. Harding deals with some aspects of the theory of fractal transformations. The authors discuss tri-affine iterated function systems on R^3 and construct 3D fractal homeomorphisms. Several examples are presented, applied to 3D graphical data sets.

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In Chapter 5, J. Barral and J. Peyrire discuss random multiplicative cascade processes, introduced by B. Mandelbrot as models exhibiting statistical features analogous to those observed empirically in velocity fluctuations of fully developed turbulent flows. The mathematical studies of non-negative and signed multiplicative cascades are performed briefly.

Chapters 6 and 7 are used for arts. J. Barrallo describes shortly the International Fractal Art Contest, while H. J. Brothers presents a marriage of music and fractal analysis. Three musical works coming from the jazz, classicism and impressionism, respectively, are examined from a fractal perspective.

In chapter 8 N. Cohen analyses the concept of fractal antenna. From technical point of view the fractal antennas are designed using fractal geometries to improve the parameters of material that can receive or transmit electromagnetic radiation within prescribed surface area or volume. A resonant characteristics of filters based on the fractal geometries are discussed also.

Chapter 9 by N. Cohen et al., gives the terse overview of fractal-based wideband invisibility cloak, which is a device that promises to revolutionize technology in a way that has never been observed before.

In chapter 10 K. Falconer discusses the local structure of fractional and multifractional Brownian, stable and multistable motions. Such processes are useful as stochastic models for phenomena that exhibit non-stationarity e.g., traffic in networks or signal processing.

Chapter 11 by R. L. Hudson, science journalist and former Wall Street Journal editor, deals with the fractal view of risk, ruin and reward, in the context of modern theories of finance.

In chapter 12 K. E. Ellis et al. provide a survey of some recent advances in the theory of fractal zeta functions. The main objects of study are associated with the following notions, namely, α -lengths, partition zeta function, abscissa of convergence function, complex dimensions with parameter α tapestry of complex of dimensions, and counting function of the α -lengths. This chapter has, somewhat, technical character and for the theory of complex dimensions for multifractals, the reader should consult the earlier works of M. L. Lapidus et al.

In chapter 13 N. Lesmoir-Gordon describes briefly the four films on fractal geometry, all directed and produced for a general audience by author. After this chapter, D. Martin, American comedian and humorist, presents the palindromic poem prepared as the project for M Frame's fractal geometry class in Yale.

Many systems in human activity generate economic time series with complex behavior. In chapter 15 W. Martino and M. Frame present the results of multifractal testing of three sets of financial data: daily closing prices for IBM, GE and Google stock. A family of multifractal measures, together with the curves corresponding to these measures are calculated via generalised dimensions and $f(\alpha)$ spectra.

Chapter 16 by N. Neger and H. J. Brothers describes briefly the didactic aspects of B. Mandelbrot's activity. There are many reasons for using fractals as pedagogical material: the wide range of fractal applications, the aesthetical aspects, the ubiquity of computers that allows the student to handle concepts previously reserved to professional mathematicians, etc. (see also <http://classes.yale.edu/fractals>; <http://www.mandelbrot.org>).

Chapter 17 by E. O'Daly gives the terse overview of some fractal case studies in architecture design. A few examples of historic buildings that exhibit fractal forms are described and commented.

"The Ising model is a crude attempt to simulate a "domain" in a ferromagnetic substance" (K. Huang, Statistical Mechanics, J Wiley and Sons, Inc., 1987, p. 341). In chapter 18 V. Pipiras and M. S. Taququ study the long-range dependence of the 2D Ising model at critical temperature. Some remarks concerning the conformal invariance scaling limits of interfaces in 2D Ising models, as well as on other lattice and growth models in random fractals context, are also sketched.

Chapter 19 by P. Renz is about the publishing story of "The Fractal Geometry of Nature". This story is presented from two perspectives: historical and technical. I will quote two extremely various opinions. First: "Exceedingly interesting book" (M.S. Longuet-Higgins, JFM, vol. 92, no. 1, 1979) and the second: "One notable difference between fractal geometry and calculus is that fractal geometry has not solved any problems" (S. Krantz, Math. Int., vol. 11, no. 4, 1989).

Lévy flights (or Lévy motion) refers to a class of non-Gaussian random processes whose stationary increments are distributed according to a Lévy stable distribution. In Chapter 20 M. Shlesinger presents some personal reflections on the fundamental properties of Lévy flights and drives. Lévy drives appear as statistical description for a broad class of processes in physical, chemical, biological, geophysical, or financial contexts.

Chapter 21 by Ch. Walter concentrates briefly on three fractal models in finance developed by B. Mandelbrot: α -stable motion with paretian tails, fractional brownian motion with aperiodic cycles and time changed Brownian motion with stochastic clock.

In chapter 22 E. R. Weibel describes shortly the importance of fractal concepts to the study of biological systems. Some medical contexts in which fractal scaling seems to be relevant are also outlined.

In general, allometry is any study of size and its consequences. Chapter 23 by B. J. West and D. West provides a brief account of the origin of the allometry concept and gives modern explanations of allometry relations within the fractal calculus and the entropic principles.

In chapter 24, Ch. Wuorinen, well-known American composer, describes some of the fractal characteristics of music and discusses the method of

fractal composition. As the author writes: “only music, I believe, of all the arts possesses an intrinsically fractal character”.

The theme of the chapter 25 is “Stories about Benoit (Mandelbrot)”. A special session dedicated to the memory of B. Mandelbrot was held in The Yale University from 29 to 30 April, 2011. Twenty-one people (distinguished mathematicians, physicians, and others), provide challenging and helpful insights into this theme. I emphasize only one story: “He wouldn’t even put his hands on a keyboard” (D. Fraccia, p. 512).

In final chapter M. Frame said shortly: “Benoit (Mandelbrot) was an enormous man”!.

Overall, I think this is an excellent book and I highly recommend this book to everyone with an interest in science, not only mathematics, whether they are professional researchers, graduate or undergraduate students or teachers.

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